Neutral stability height correction for ocean winds

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Abstract

Adjusting ocean wind observations to a standard height, usually 10 m, requires the use of a boundary layer model, and knowledge of the thermodynamical variables. Height adjustment is complicated by the fact that a necessary parameter, the roughness height, cannot be given in a closed form solution. If only the wind and reporting height are known, the best that can be done is to assume neutral stability. The determination of roughness height is analyzed and a simple approximation is given that is accurate for winds in the range of $1 - 30\ \mathrm{m\,s^{-1}}$, for neutral stratification. This approximation would also be an excellent initial estimate for a Newton iteration to determine the roughness height precisely, whether or not neutral stability is assumed.

1. Introduction

Adjusting ocean wind observations to a standard height, usually 10 m requires the use of boundary layer model, and knowledge of the thermodynamical variables. Whichever PBL model is used, it is necessary to iterate to solve the constant flux layer equations. This is due to the fact that z_0, the roughness length, is an implicit function of the model variables over the oceans. The Charnock formula states that over the ocean z_0 and the surface stress magnitude $|\tau|$ are linearly related by

$$z_0 = \frac{a}{\rho g}|\tau|. \tag{1}$$

Here $a = 0.032$, $\mathrm{g} = 9.81$ m s^{-2}, and ρ, the density of air, is assumed to be constant for the range of heights considered and equal to its surface value. However the surface stress determined from

$$\tau = -\rho C_d |V| V \tag{2}$$

depends on z_0 through the neutral drag coefficient, and in the unstable case on the similarity function, usually denoted $f(Ri)$ where Ri is the Richardson number. In Eq. (2) V is the vector wind at some height z, $|V|$ is the magnitude of the vector wind, the directions of the wind and stress vector are assumed to be parallel for the range of heights considered, and the drag coefficient C_d is given by the product of the similarity function $f(Ri)$ and the neutral drag coefficient is given by

$$C_{dn} = \left[\frac{k}{\log\left(\frac{z}{z_0}\right)}\right]^2. \tag{3}$$

Here the von Kármán constant $k = 0.4$. For neutral stratification, $Ri = 0$, and $f(Ri) = 1$. See Hoffman and Louis (1990) for details.

NWP models usually "cheat" and use the value of τ of the previous time step to find z_0 through the Charnock formula. Actually using old values to evaluate the dissipative terms can be a good policy as this can reduce computational instability. But outside of a model we must calculate z_0 implicitly. For this purpose we substitute the absolute value of Eq. (2) into Eq. (1) to obtain

$$z_0 = (a/g)C_d V^2 \equiv h(z_0; z, Ri). \tag{4}$$

To solve Eq. (4) we must iterate. To begin the process, Hoffman and Louis (1990) estimated C_d as a linear function of $|V|$ and then obtained the initial estimate of z_0 from the Charnock relationship.

Then Eq. (4), $z_0 = h$, is iterated. This converges to a good approximation within a few iterations. It is then possible to switch to a Newton iteration to solve $z_0 - h = 0$. The Newton method requires the partial derivative of h with respect to z_0. This can be evaluated using the tangent linear code corresponding to the calculation of h by setting all inputs to zero except for that corresponding to z_0, which is set to unity.

2. Height correction for ocean winds

Knowing z_0 is equivalent to knowing the stress, and we can then solve Eq. (4) for $|V|$ at any height, z, if only we know the Richardson number. In particular, Eq. (4) states that $C_d^{\frac{1}{2}}|V|$ is conserved as we vary the height z. However Ri depends on knowing the stratification of the boundary layer. If only the wind and reporting height are known, the best that can be done is to assume neutral stability. This allows us to determine the 10 m wind speed $|V_{10}|$ from an observation at some other height according to:

$$|V_{10}| = \left[\frac{\log(10/z_0)}{\log(z/z_0)}\right]|V|. \tag{5}$$

Note that once z_0 is determined, the neutral wind U, which is defined by

$$\tau = -\rho C_{dn}|U|U \tag{6}$$

is easily determined.

3. Calculation of z_0 under neutral conditions

To apply Eq. (5) we still need to determine z_0. Here we demonstrate a simple approximation. The motivation is that under neutral conditions, for some fixed height, we expect wind speed, surface stress, and roughness height to all increase together. (Differentiating Eq. (10), given below, we obtain

$$\frac{d|V|}{dz_0} = \left(\frac{g}{ak^2}\right)^{\frac{1}{2}} z_0{}^{-\frac{1}{2}}\left[\frac{1}{2}\log\left(\frac{z}{z_0}\right) - 1\right]. \tag{7}$$

Table 1: Sample calculations based on Eq. (10). The z_0 values are equal to 2^{-k} but have been multiplied by 10^6 for presentation in this table. The $|V_{10}|$ values are in $\mathrm{m\,s^{-1}}$ and are calculated using Eq. (10). The ratios in columns 4 and 5 are equal to the term in square brackets in Eq. (5). The last column contains the estimated value of z_0 from Eq. (12) for the values of $|V_{10}|$ listed.

k	z_0	$\lvert V_{10}\rvert$	$\lvert V_{10}/V_4\rvert$	$\lvert V_{10}/V_{19.5}\rvert$	z_0(calc)
15	31	3.1	1.08	0.950	35
11	488	9.6	1.10	0.937	493
7	7813	27.7	1.15	0.915	5813

Thus $d|V|/dz_0/ > 0$ provided $z > e^2 z_0$. This holds for wind speeds less than hurricane strength and heights of several meters or more.) The suggestion then is that z_0 should be a monotonically increasing function of $|V|$, and interpolation into a look-up table, or a simple fit to a set of exact values should work.

For this investigation it is convenient to define

$$y = \log(z/z_0) \quad \text{and} \quad z_0 = ze^{-y}. \tag{8}$$

Then Eq. (4) specialized for neutral stability may be written as

$$y^2 e^{-y} = \frac{ak^2}{gz} V^2 \equiv \gamma. \tag{9}$$

Given V and z, Eq. (9) determines γ. Now if we can tabulate or model y as a function of γ, then V and z would determine γ, then y, and finally z_0 using Eq. (8).

Of course, given y, it is trivial to calculate γ. Therefore given z_0 and z we immediately have

$$V = \left(\frac{g}{ak^2}\right)^{\frac{1}{2}} z_0{}^{\frac{1}{2}} \log\left(\frac{z}{z_0}\right). \tag{10}$$

A few sample calculations using Eq. (10) are presented in Table 1 for heights of 4, 10, and 19.5 m. Values of z_0 given by 2^{-k} are used here and in the figures presented below.

In Table 1 we see that z_0 varies by more than two orders of magnitude as wind speed varies from 3 to 30 $\mathrm{m\,s^{-1}}$. Over this range of wind speed, the correction factors for determining $|V_{10}|$ vary

by roughly 5%. This variation is the same order of magnitude as the corrections, and is therefore worth accounting for.

Figure 1a plots y as a function of $\log(\gamma)$ for $k = 0, \ldots, 30$. Clearly a linear fit will work well over most the range of γ. This is not unexpected since according to Eq. (9) $\log\gamma = -y + 2\log y$. As k increases, z_0, $|V_{10}|$, and γ decrease, while y increases. For example, for $k = 0$, $z_0 = 1$ m and $|\mathrm{V}_{10}| = 100$ m s^{-1}, while for $k = 30$, $z_0 \approx 10^{-9}$ m and $|\mathrm{V}_{10}| = 0.03$ m s^{-1}. Fitting points for $k \geq 6$, corresponding to $|\mathrm{V}_{10}| \leq 35$ m s^{-1} (where the extreme point included is marked by the vertical line in the plots) we find that

$$y = c_0 + c_1 \log(\gamma) \tag{11}$$

with $c_0 = 3.7$ and $c_1 = -1.165$.

Combining Eq. (8), Eq. (9), and Eq. (11), we obtain our estimate of z_0

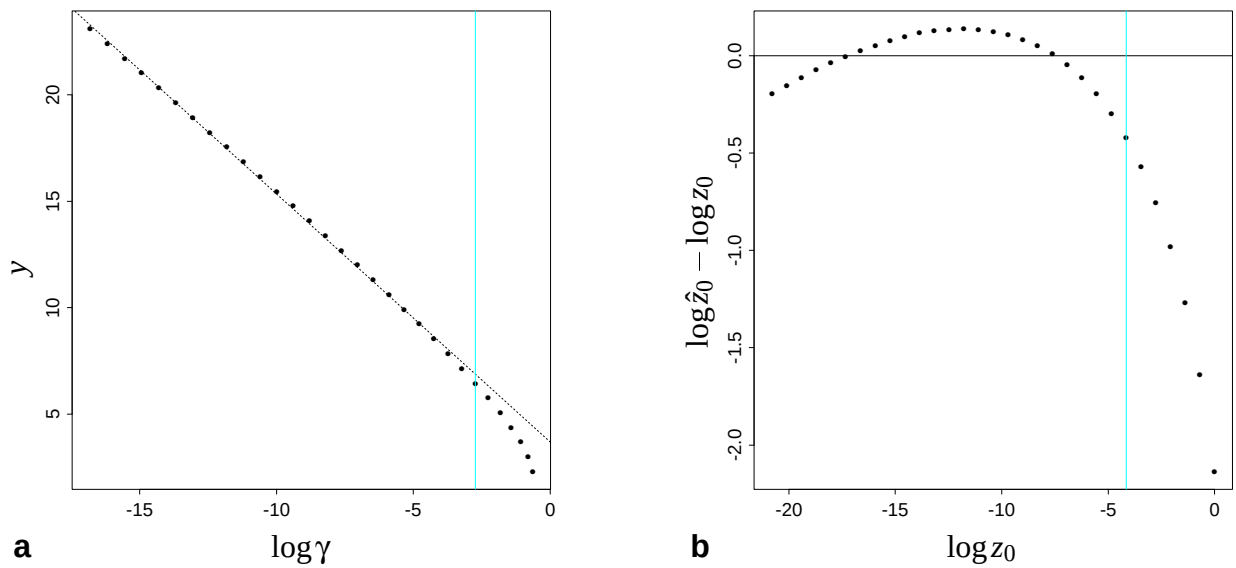
$$\hat{z}_0 = z \exp[-(c_0 + c_1 \log[(ak^2V^2)/(gz)])]. \tag{12}$$

Figure 1b shows the error of the fit in log space. Values calculated using Eq. (12) are shown in the last column of Table 1 for the three cases listed. The differences are not tiny, but when we recalculate the ratios of the wind speeds in columns 4 and 5 of the table, the results are nearly the same. Using the same precision as in the table the values are the same, except for last row where they differ in the last significant figure (the new values are 1.14 and 0.917 respectively).

4. Concluding remarks

An approximation for roughness height (Eq. (12)) is given that is accurate for winds in the range of $1 - 30$ m s^{-1} under the assumption of neutral stability. Values of z_0 that vary with wind speed should be used in correcting ocean winds to a standard height. Typical correction are in the range of 5-15%, so assuming a single value will incur errors of a few per cent. Our approximation would also be an excellent initial estimate to begin a Newton iteration to determine the roughness height precisely, whether or not neutral stability is assumed.

Figure 1: Fitting $y = \log(z/z_0)$ as a function of $\gamma = (ak^2/gz)V^2$. (a) The linear fit to values of y and $\log\gamma$ for $k = 6,\ldots,30$ is plotted as a dotted line. The data values are plotted as dots. (b) Log residuals for Eq. (12) fit of z_0. For this calculation we define the true values of z_0 and then $|V|$ for the chosen value of z (10 m) using Eq. (10), and then use these values of $|V|$ and z in Eq. (12) to determine $\hat{z}_0$. The vertical lines identify $k = 6$.



References

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