Coupled atmosphere-ocean data assimilation in the presence of model error

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Introduction

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Coupled DA faces many challenges, here we focus on the specific problem of **model error** which may be amplified via the coupling.

- Model error in the coupled system **restricts the window length** which can be used with 4D-Var to something shorter than the optimal window length in an uncoupled Ocean DA scheme.
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Aim of work is to understand how **different coupling strategies** react as window length is extended and the model error becomes more significant.

Coupling strategies (recap)

Each method is based on incremental 4D-Var. In each case the first guess comes from a coupled forecast.

Strongly coupled DA: uses the coupled model in both the inner and outer loops.

Weakly coupled DA: uses the coupled model in the outer loop but the inner loop is uncoupled.

Uncoupled DA: uses the uncoupled models in both the outer and inner loops. BCs at the interface are prescribed externally.

Model error in 4D-Var

- In 4D-Var have the assumption that the background state, $\mathbf{x}^{\mathrm{b}}$, and the observations, $\hat{\mathbf{y}}$, are consistent with

$$\mathbf{x}^{\mathrm{b}} \sim N(\mathbf{x}_0^{\mathrm{t}}, \mathbf{B}) \text{ and } \hat{\mathbf{y}} \sim N(\hat{\mathbf{y}}^{\mathrm{t}}, \hat{\mathbf{R}})$$

$$\text{and } \hat{\mathbf{y}}^{\mathrm{t}} = \hat{\mathcal{H}}(\mathbf{x}_0^{\mathrm{t}})$$

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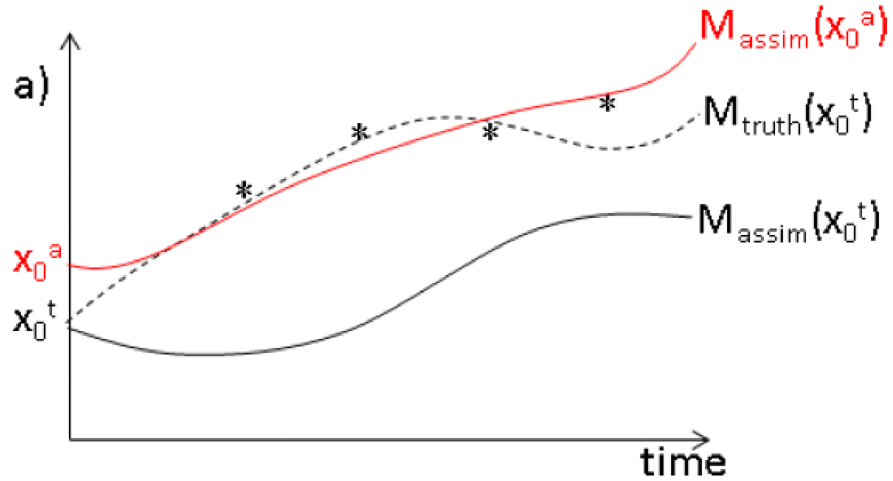
- However if model error becomes significant then this last assumption breaks down, and instead:

$$\begin{aligned} \hat{\mathbf{y}}^{\mathrm{t}} &= \hat{\mathcal{H}}^{\mathrm{t}}(\mathbf{x}_0^{\mathrm{t}}) \\ &= \hat{\mathcal{H}}(\mathbf{x}_0^{\mathrm{t}}) + \epsilon^{\hat{\mathcal{H}}} \quad \text{where} \quad \epsilon^{\hat{\mathcal{H}}} \in \mathbb{R}^{\hat{p} \times 1} \end{aligned}$$

Model error in 4D-Var



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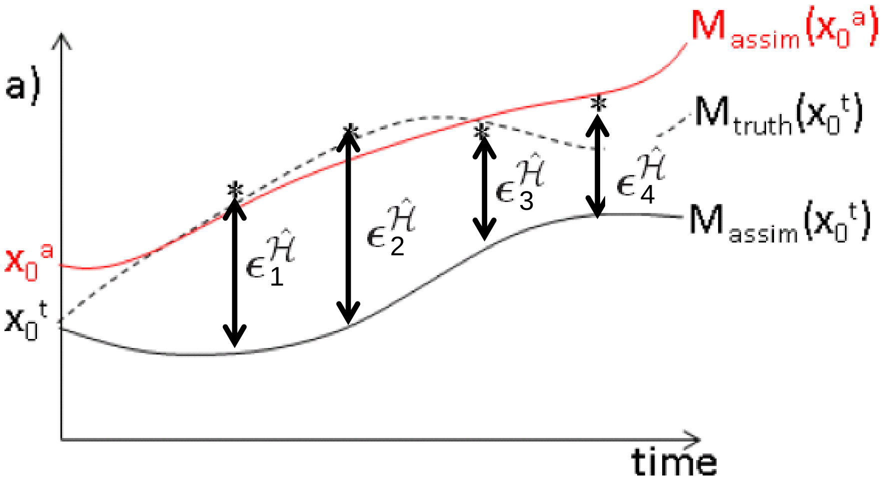


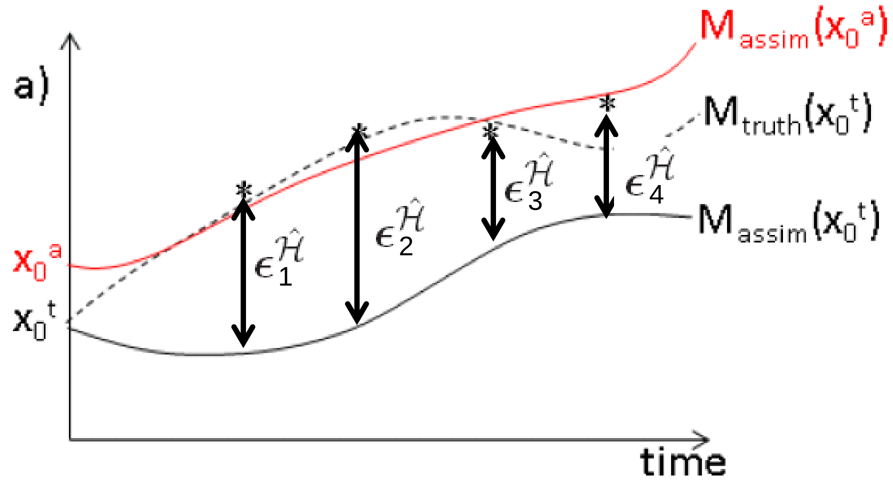
If the model error is unaccounted for it has the effect of increasing the analysis error covariances

$$E[\epsilon^{\mathrm{a}}(\epsilon^{\mathrm{a}})^{\mathrm{T}}] = \mathbf{P}^{\mathrm{a}}_{\mathrm{nm}} + \mathbf{K}E[\epsilon^{\hat{\mathcal{H}}}(\epsilon^{\hat{\mathcal{H}}})^{\mathrm{T}}]\mathbf{K}^{\mathrm{T}}$$

where $\mathbf{P}^{\mathrm{a}}_{\mathrm{nm}}$ is the analysis error covariance matrix when no model error is present.

where $\mathbf{K} \in \mathbb{R}^{n\times\hat{p}}$ is the Kalman gain matrix.

Model error in 4D-Var



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where $\mathbf{P}^{\mathrm{a}}_{\mathrm{nm}}$ is the analysis error covariance matrix when no model error is present.

And if the model error is biased then the analysis error will also be biased

$$E[\boldsymbol{\epsilon}^{\mathrm{a}}] = \mathbf{K}E[\boldsymbol{\epsilon}^{\hat{\mathcal{H}}}].$$

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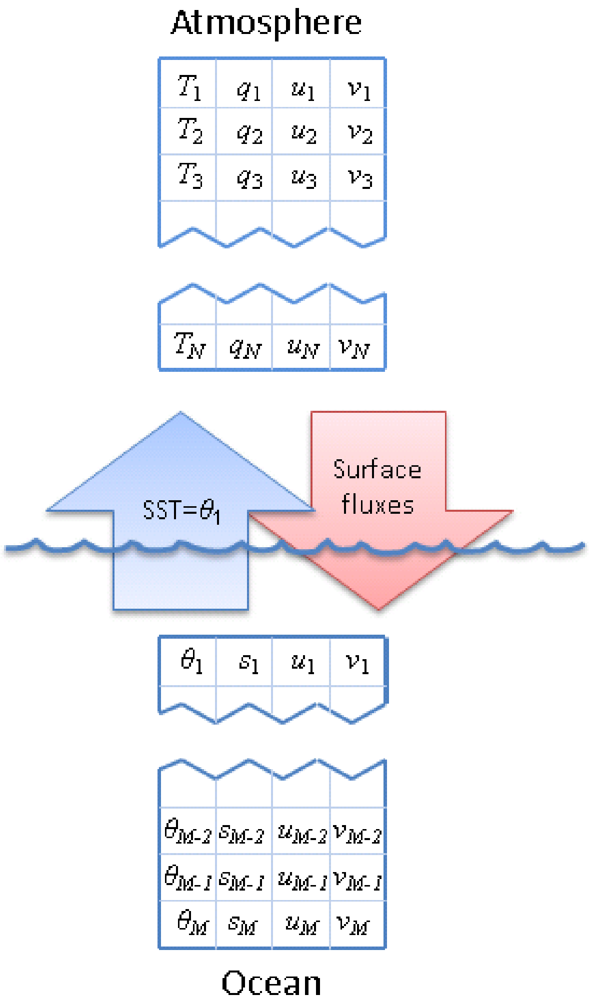
Strongly coupled $\epsilon^{\hat{\mathbf{H}}} = \hat{\mathcal{H}}^{\mathrm{t}}(\mathbf{x}_0^{\mathrm{t}}) - \hat{\mathcal{H}}^{\mathrm{c}}(\mathbf{x}_0^{\mathrm{g}}) - \hat{\mathbf{H}}^{\mathrm{c}}|_{\mathbf{x}^{\mathrm{g}}}(\mathbf{x}_0^{\mathrm{t}} - \mathbf{x}_0^{\mathrm{g}})$

Weakly coupled $\epsilon^{\hat{\mathbf{H}}} = \hat{\mathcal{H}}^{t}(\mathbf{x}_0^{\mathrm{t}}) - \hat{\mathcal{H}}^{\mathrm{c}}(\mathbf{x}_0^{\mathrm{g}}) - \hat{\mathbf{H}}^{\mathrm{uc}}|_{\mathbf{x}^{\mathrm{g}}}(\mathbf{x}_0^{\mathrm{t}} - \mathbf{x}_0^{\mathrm{g}})$

uncoupled $\epsilon^{\hat{\mathbf{H}}} = \hat{\mathcal{H}}^{t}(\mathbf{x}_0^{\mathrm{t}}) - \hat{\mathcal{H}}^{\mathrm{uc}}(\mathbf{x}_0^{\mathrm{g}}) - \hat{\mathbf{H}}^{\mathrm{uc}}|_{\mathbf{x}^{\mathrm{g}}}(\mathbf{x}_0^{\mathrm{t}} - \mathbf{x}_0^{\mathrm{g}})$

Model set up
- generation of model error



The model error is assumed to be complex and from multiple sources.

<u>Atmosphere</u>

Assimilation model has missing physics and a bias in the large scale forcing.

<u>Ocean</u>

Assimilation model has perturbed diffusion parameters.

Results are shown for the July 2014 case study for a point in the NW Pacific.

Model error in the atmosphere (no DA)

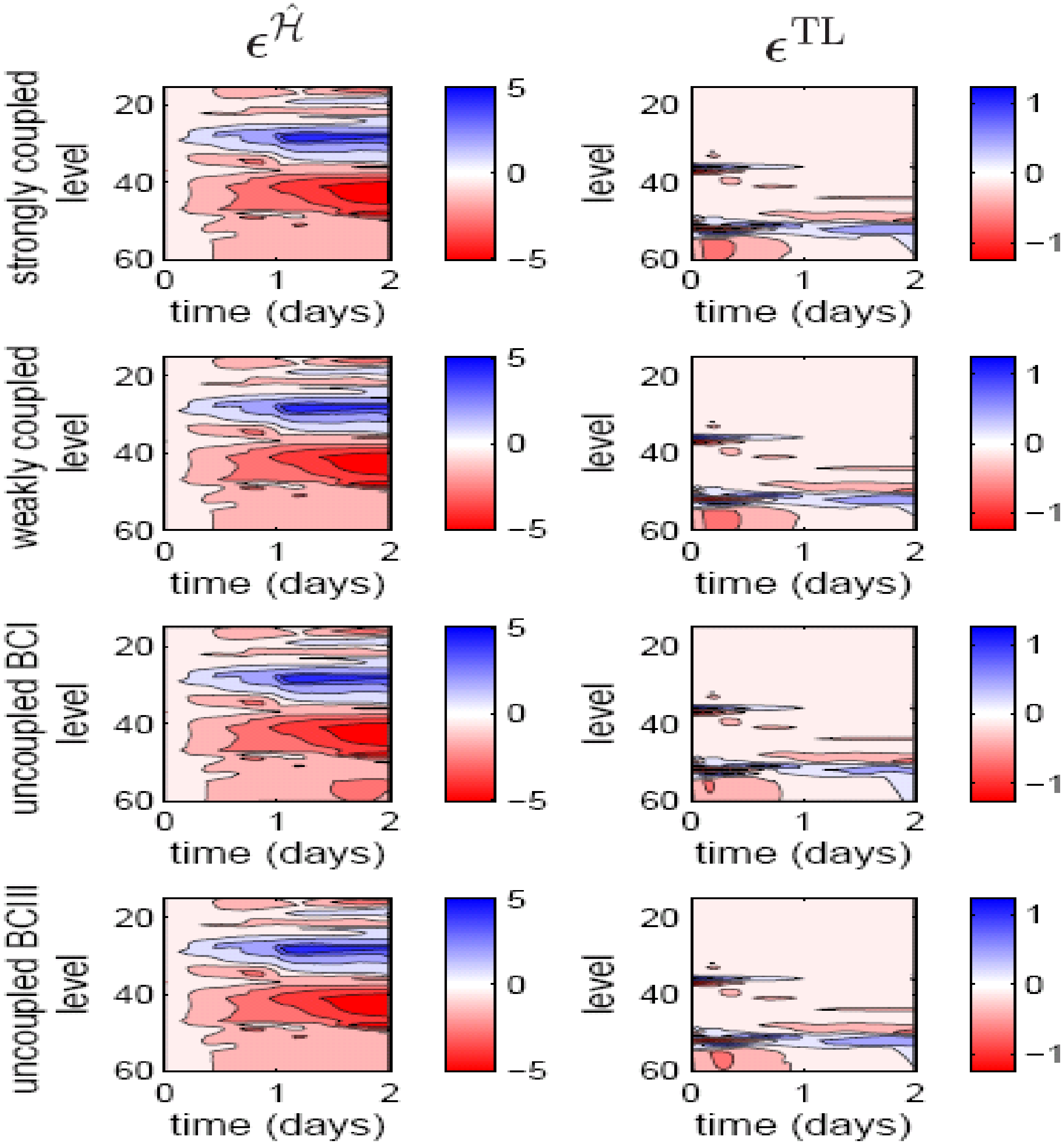


Figure: Model error over 2 days for atmospheric temperature

Uncoupled DA with a poor estimate of the BCs

Uncoupled DA with a good estimate of the BCs

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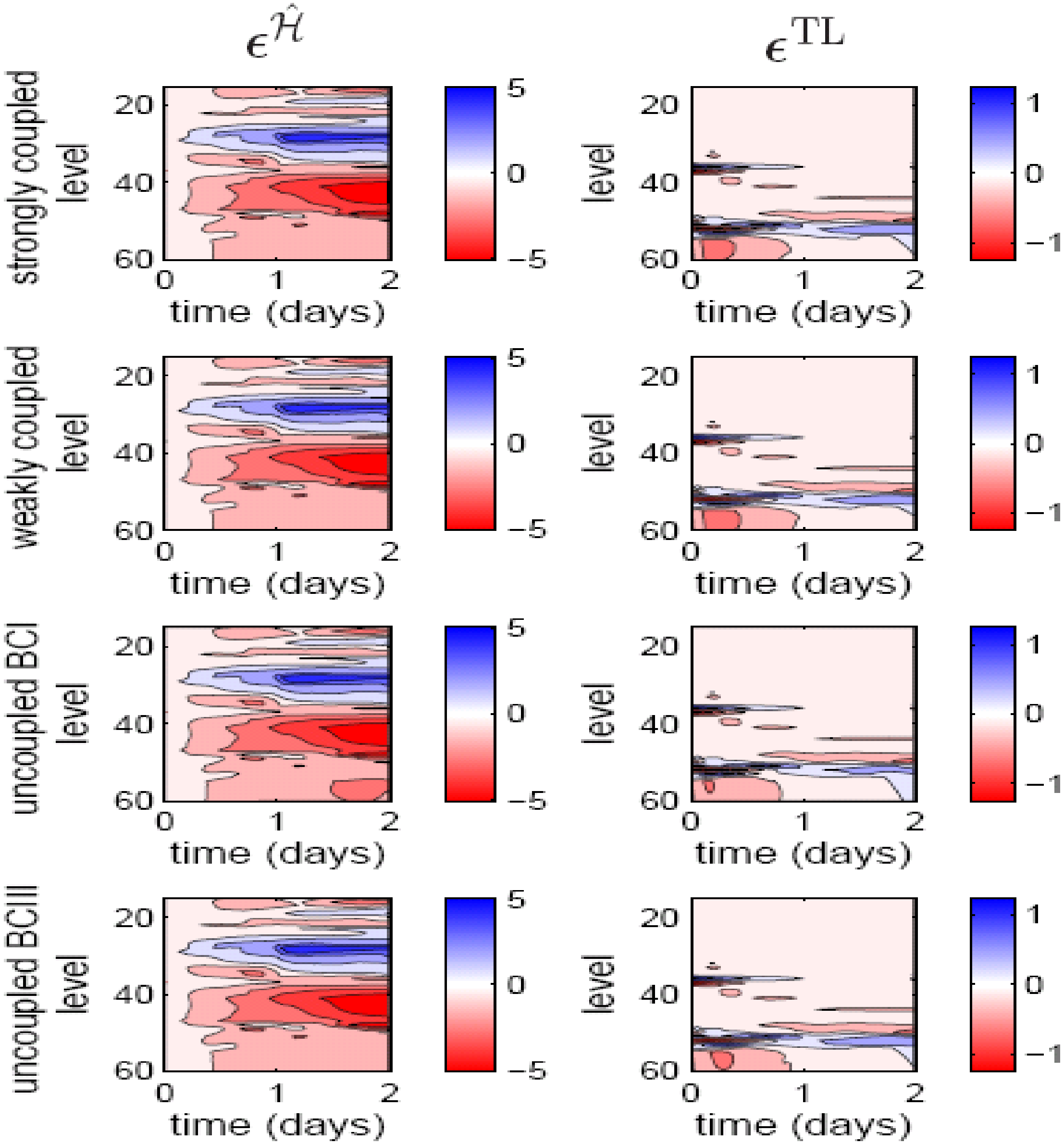

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Observation error standard deviation is 1K.

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Model error in the ocean (no DA)

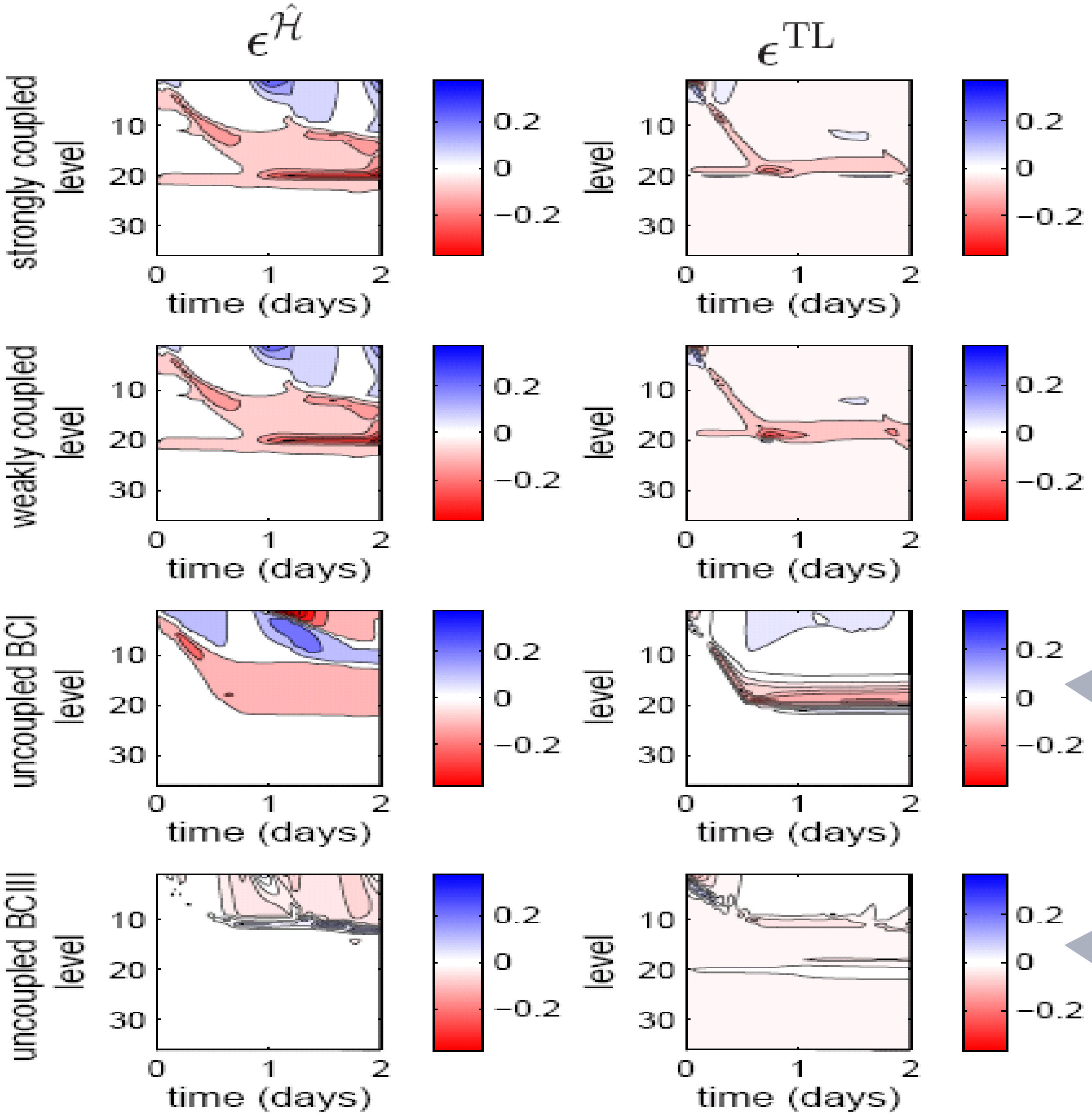

Figure: Model error over 2 days for oceanic temperature

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Model error in the ocean (no DA)

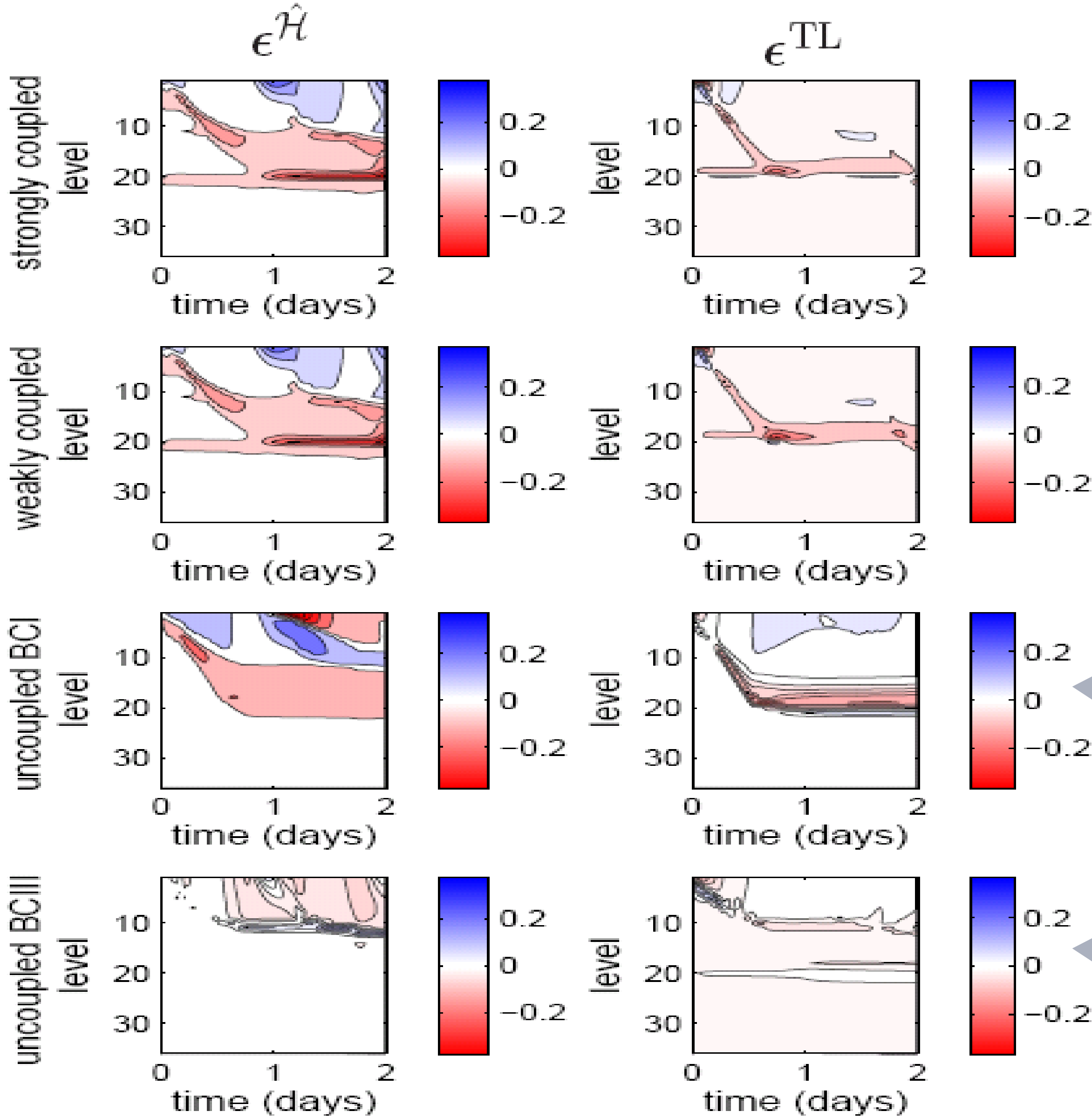

Figure: Model error over 2 days for oceanic temperature

Observation error standard deviation is 0.1K.

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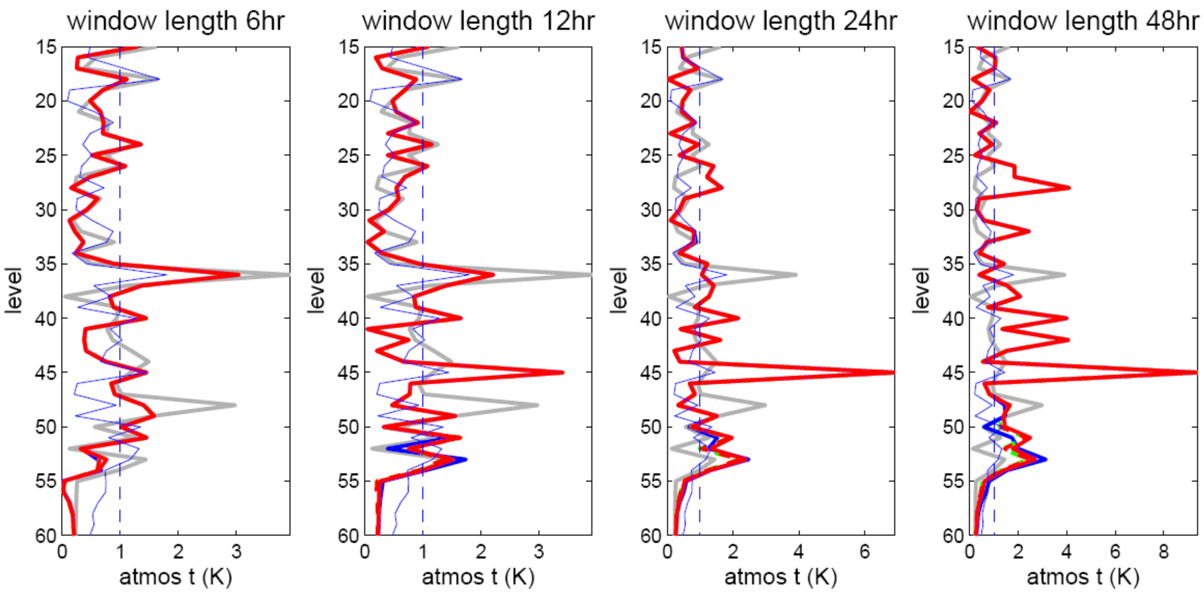
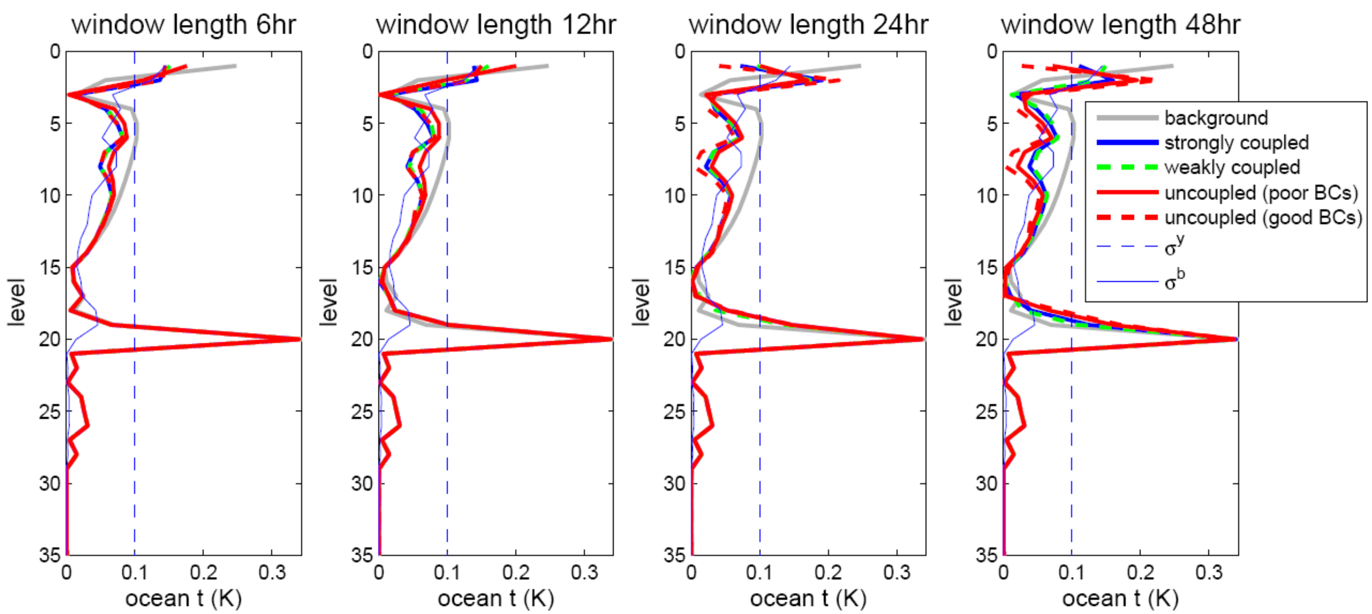
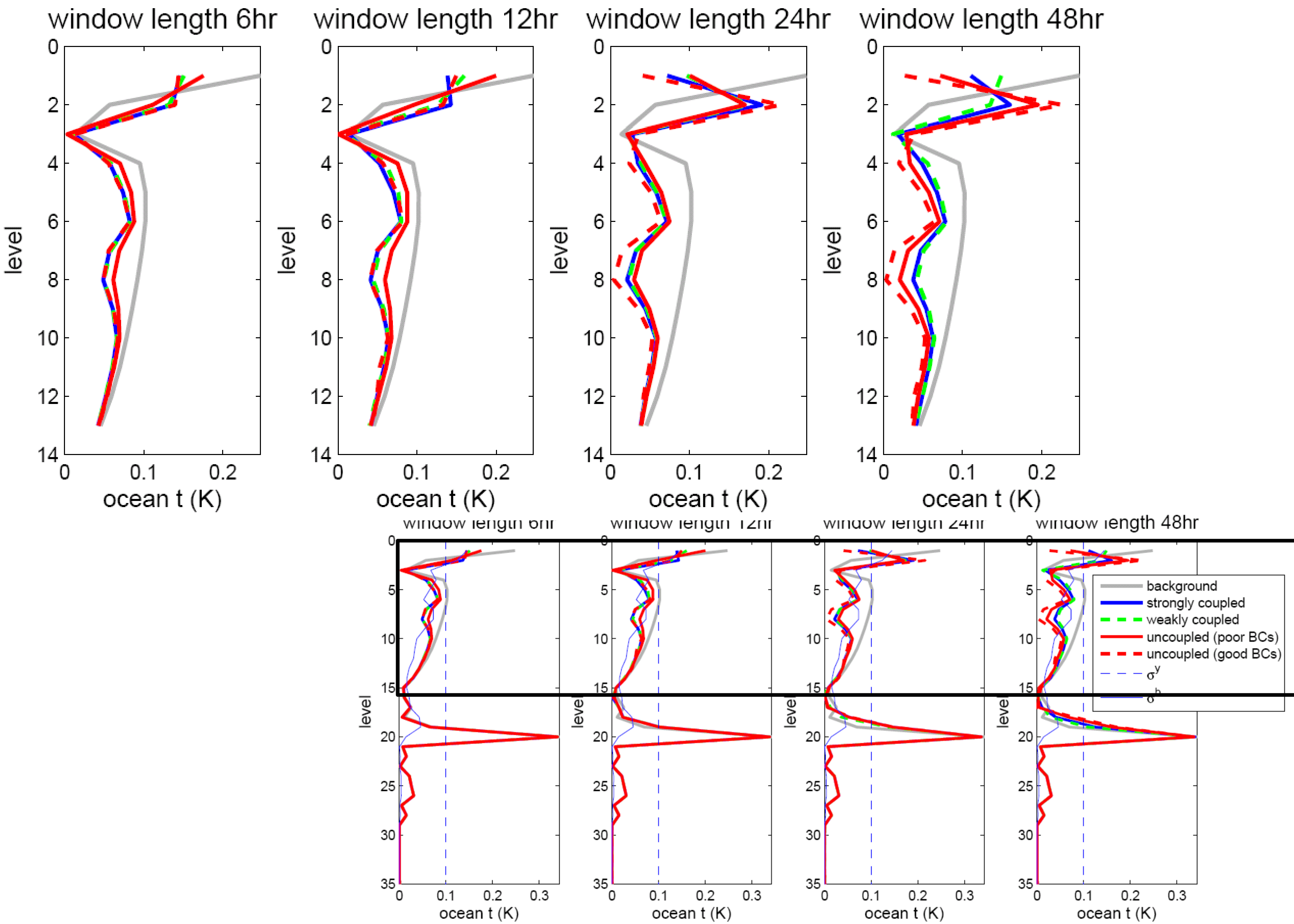
Absolute analysis error ($|x^a - x^t|$)



Figures: Absolute error in temperature at the initial time, in the atmosphere (left) and ocean (below).

- Atmospheric observations every 3 hours, ocean observations every 6 hours.
- Each level is observed.
- B is diagonal





window length 6hr
window length 12hr
window length 24hr
window length 48hr
level
ocean t (K)
background
strongly coupled
weakly coupled
uncoupled (poor BCs)
uncoupled (good BCs)
σ^y
σ^b

Forecast error in the atmosphere ($M^t(x^t)$-$M(x^a)$)

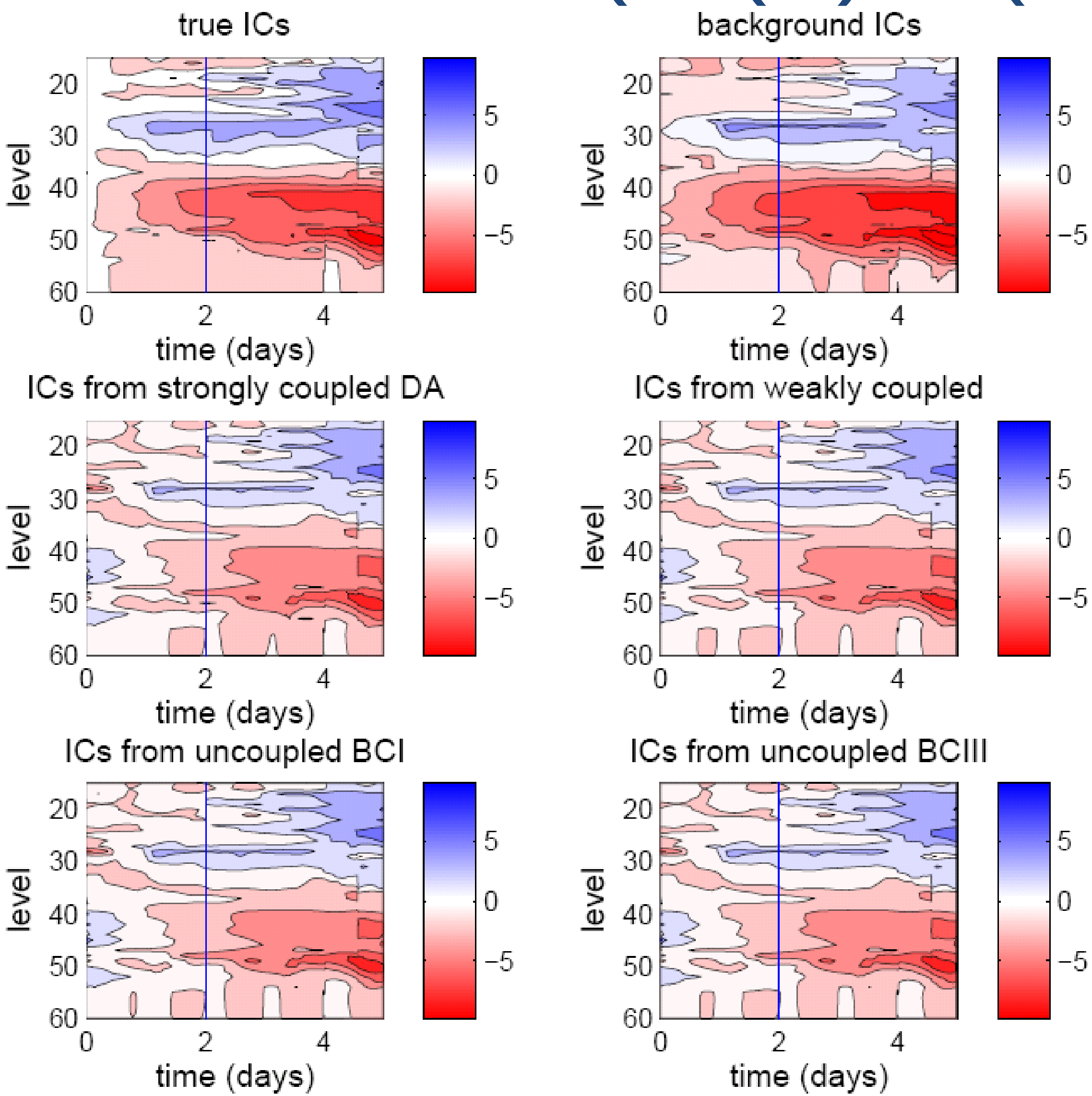


Figure: Forecasts of atmospheric temperature <u>using coupled model</u> initialised using different analyses computed using a <u>2 day assimilation window</u>

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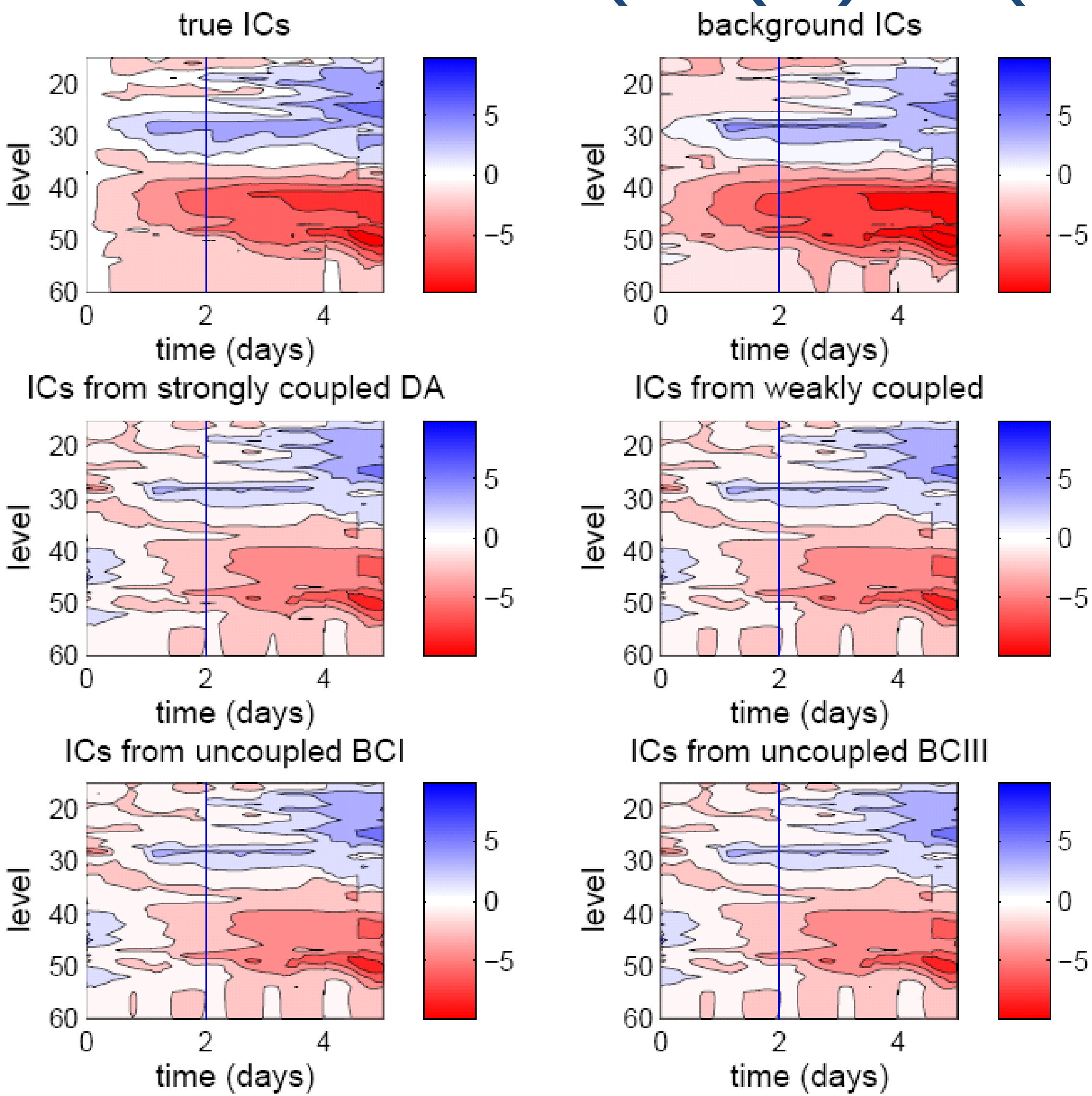

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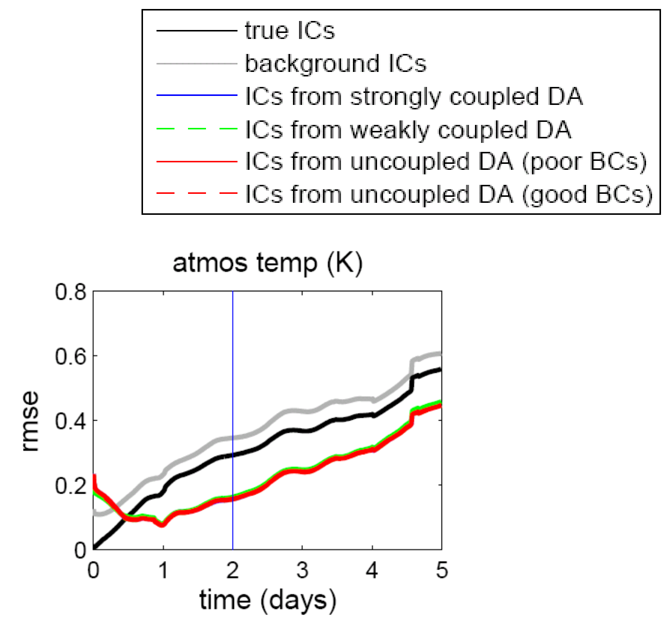
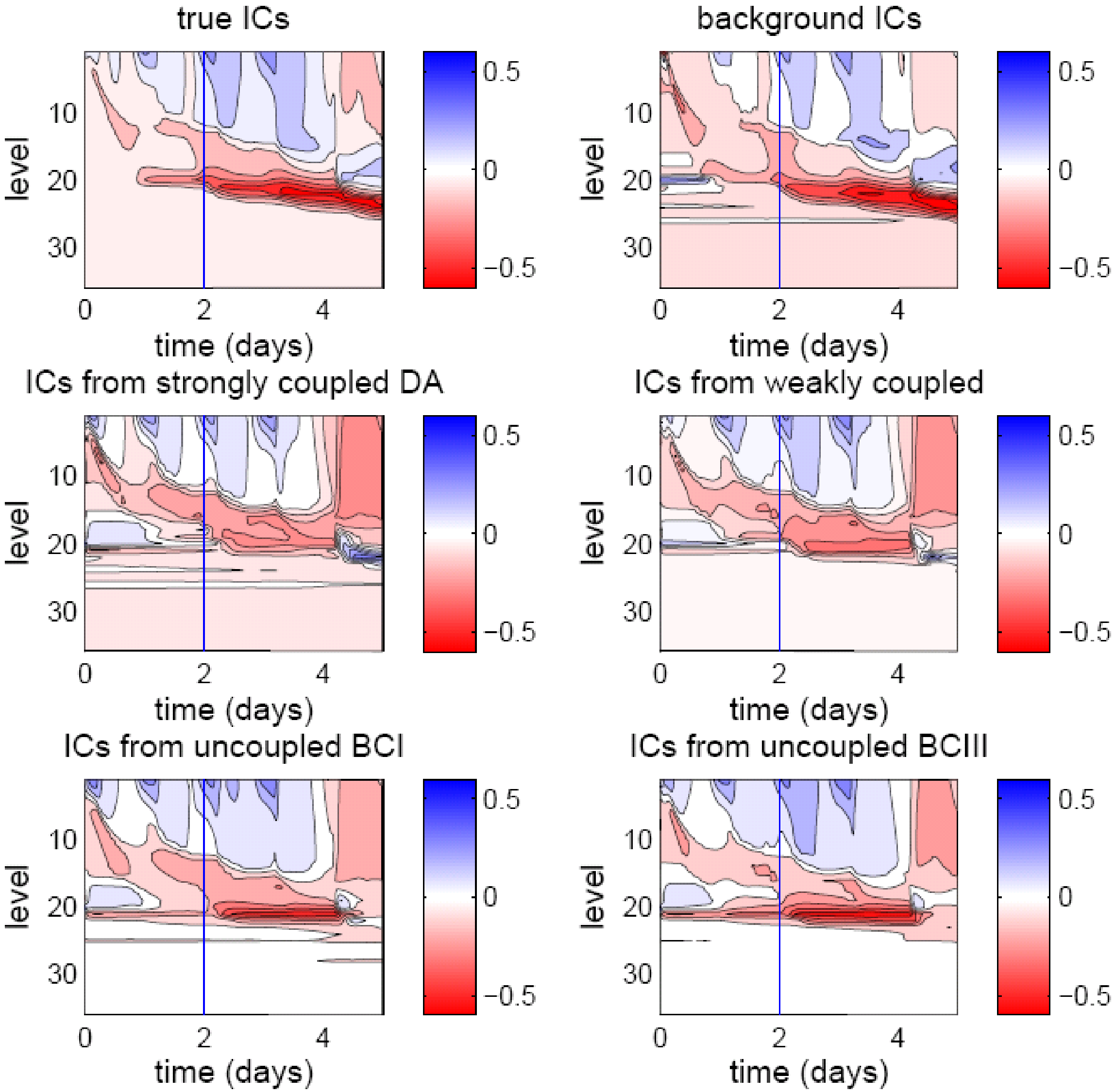


Forecast error in the ocean ($M^t(x^t)$-$M(x^a)$)



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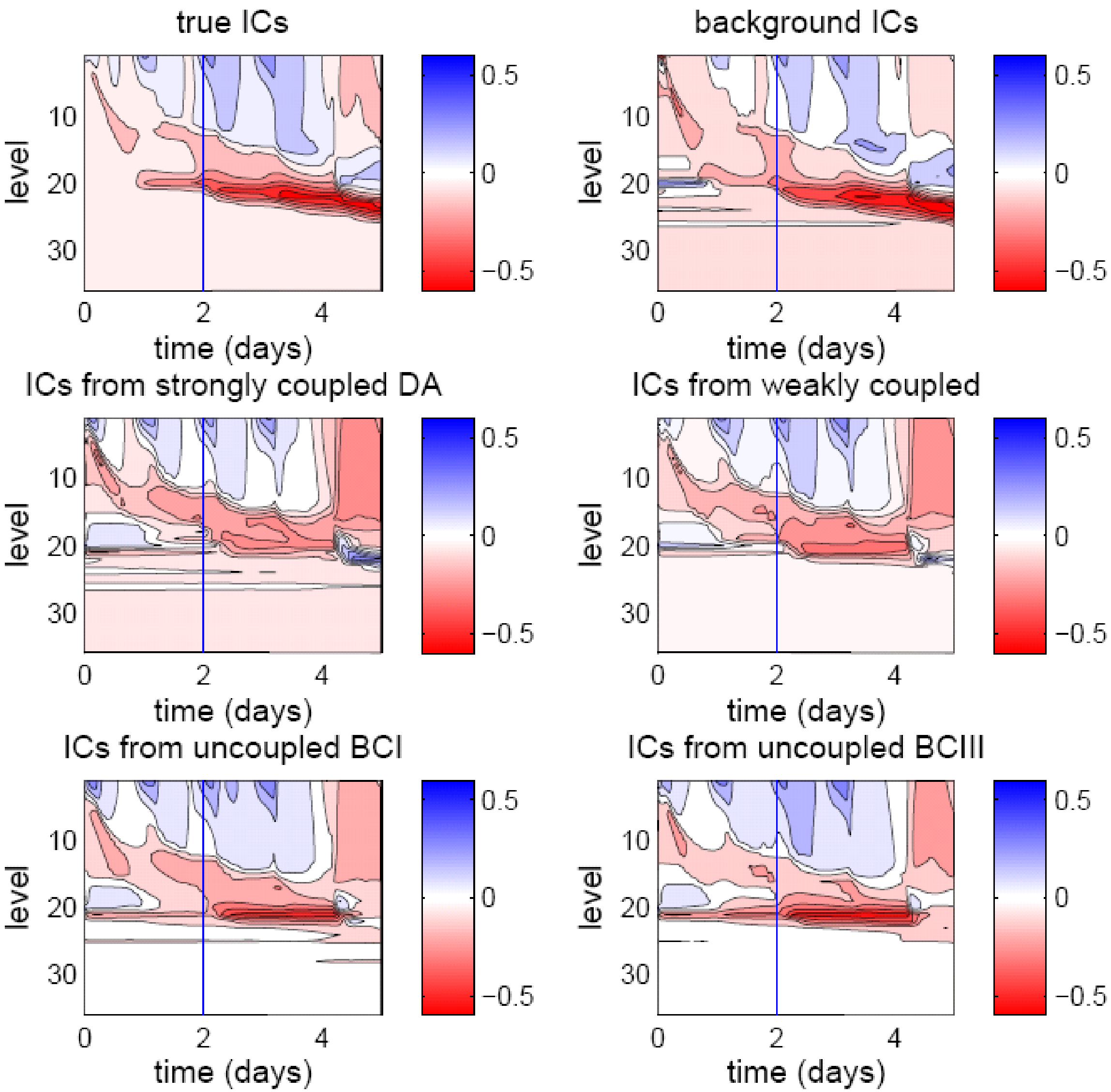

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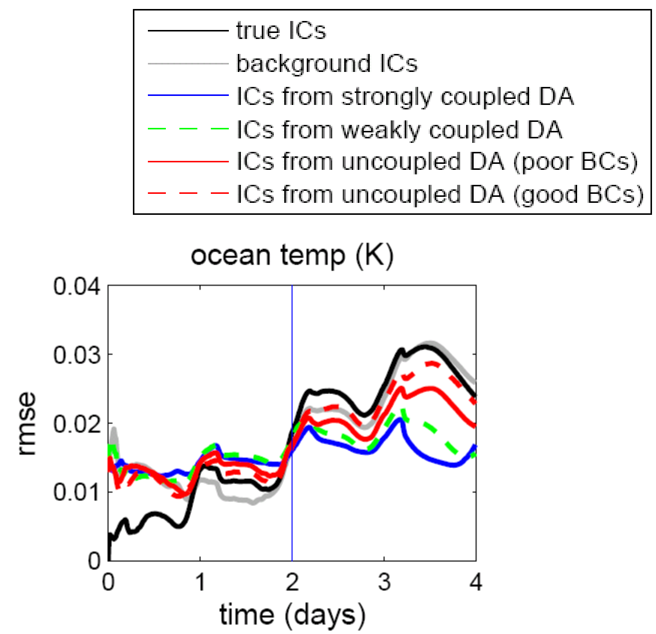

Summary- 2 day window length

Strongly coupled DA

•poor analysis due to observations being inconsistent with assimilation model

•Produces a more balanced initial state and results in the best forecast beyond a day.

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Strongly coupled DA	Uncoupled DA
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Strongly coupled DA	Uncoupled DA	Weakly coupled DA
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Conclusions

- The coupling of the atmosphere and ocean can amplify the presence of model error.
- The effect of model error on the analysis depends on the coupled DA scheme used.
- Strongly coupled DA has been shown to be able to provide an analysis consistent with the forecast model at the expense of the accuracy of the ocean analysis.
- To improve the utility of strongly coupled DA need to be able to account for model error in the assimilation to allow for the window length to be extended.

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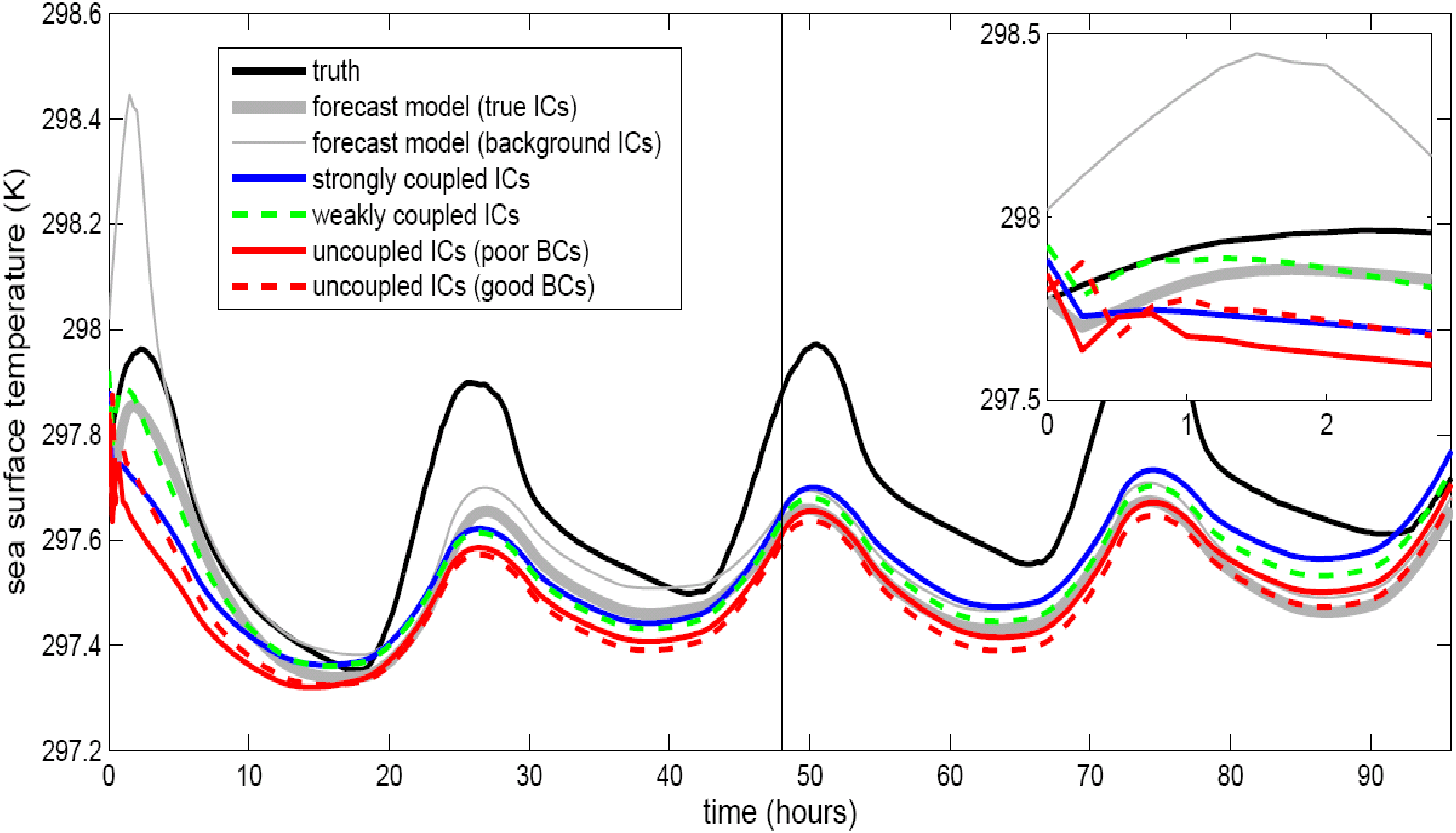
Thank you for listening

Initialisation shock



Initialisation shock- reduced observations