An idealised fluid model for inexpensive DA experiments and its relevance for NWP

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DA: from large- to convective-scale

High-resolution (convective-scale) NWP models are becoming the norm

- more dynamical processes such as convection, cloud formation, and small-scale gravity waves, are resolved explicitly

DA techniques need to evolve in order to keep up with the developments in high-resolution NWP

- breakdown of dynamical balances (e.g., hydrostatic and semi/quasi-gestrophic) at smaller scales
- strongly nonlinear processes associated with convection and moisture/precipitation
- move towards ensemble-based methods

Using idealised models

It may be unfeasible, and indeed undesirable, to investigate the potential of DA schemes on state-of-the-art NWP models. Instead idealised models can be employed that:

- capture some fundamental processes
- are computationally inexpensive to implement
- allow an extensive investigation of a forecast/assimilation system in a controlled environment

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'Toy' models:

- Lorenz (L63, L95, L2005, ...)
- BV/QG models (Bokhove et al., poster this workshop)
- simplified NWP models

Using idealised models: approach

1. Describe a physically plausible idealised model and implement numerically.
   - based on the shallow water equations (SWEs).
   - compare dynamics of the modified model to those of the classical shallow water theory
2. Ensemble-based DA - relevant for convective-scale NWP?
   - initial perturbations to represent forecast error
   - "tuning" the observing system and the observational influence diagnostic
3. Current/future work and ideas.
   - DA: a comparison with VAR
   - advanced numerics: non-negativity of 'rain'
   - other fluid dynamical models
   - which characteristics of NWP can we seek to replicate in idealised models?

1. SWEs: an extension

Aim: modify the SWEs to include more complex dynamics relevant for the 'convective-scale', extending the model employed by Würsch and Craig (2014).

- convective updrafts - artificially mimic conditional instability (positive buoyancy)
- idealised representation of precipitation, including source and sink.
- contain switches for the onset of convection and precipitation - realistic (and highly nonlinear) features of operational NWP models.

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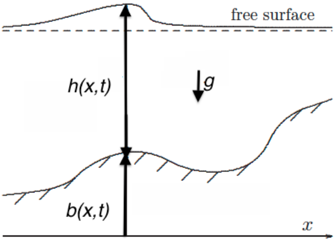
2D rotating SWEs on an f-plane with no variation in the y-direction ($\partial_y = 0$):

$$\partial_t h + \partial_x(hu) = 0,$$
$$\partial_t(hu) + \partial_x(hu^2 + p(h)) - fhv = -gh\partial_x b,$$
$$\partial_t(hv) + \partial_x(huv) + fhu = 0,$$
$$\partial_t b = 0,$$

where $p(h)$ is an effective pressure: $p(h) = \frac{1}{2}gh^2$.



Modified SWEs

Ingredients:

- two threshold heights $H_c < H_r$: when fluid exceeds these heights, different mechanisms kick in and alter the classical SW dynamics.
- modifications to the effective pressure gradient (equivalently, geopotential gradient) in the momentum equation.
- extra equation for the conservation of model 'rain' to close the system.

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\begin{aligned}
&\partial_t h + \partial_x(hu) = 0,\\
&\partial_t(hu) + \partial_x(hu^2 + p(h)) + hc_0^2\partial_x r - fhv = -gh\partial_x b,\\
&\partial_t(hv) + \partial_x(huv) + fhu = 0,\\
&\partial_t(hr) + \partial_x(hur) + h\widetilde{\beta}\partial_x u + \alpha hr = 0,\\
&\partial_t b = 0,
\end{aligned}
$$

where $p(h) = \begin{cases} \frac{1}{2}gH_c^2, & \text{for } h+b > H_c,\\ \frac{1}{2}gh^2, & \text{otherwise,}\end{cases}$ and $\widetilde{\beta} = \begin{cases} \beta, & \text{for } h+b > H_r,\ \partial_x u < 0,\\ 0, & \text{otherwise.}\end{cases}$

Some theoretical aspects

- Shallow water systems are hyperbolic, and can thus be solved via a range of numerical recipes for hyperbolic syststems. What about the modified system?
- Vector formulation:

$$\partial_t \mathsf{U} + \partial_x \mathsf{F}(\mathsf{U}) + \mathsf{G}(\mathsf{U})\partial_x \mathsf{U} + \mathsf{S}(\mathsf{U}) = 0$$

- Hyperbolicity determined by eigenstructure (all eigenvalues must be real). Eigenvalues of the system are determined by the matrix:

$$\partial \mathsf{F}/\partial \mathsf{U} + \mathsf{G}(\mathsf{U}) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -u^2 - c_0^2 r + \partial_h p & 2u & c_0^2 & 0 & gh \\ -u(\widetilde{\beta} + r) & \widetilde{\beta} + r & u & 0 & 0 \\ -uv & v & 0 & u & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

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- This matrix has five eigenvalues:

$$\lambda_{1,2} = u \pm \sqrt{\partial_h p + c_0^2 \widetilde{\beta}}, \quad \lambda_{3,4} = u, \quad \text{and} \quad \lambda_5 = 0,$$

- Since $p(h)$ is non-decreasing and $\widetilde{\beta}$ non-negative, the eigenvalues are real. Hence, the modified SW model is hyperbolic.

Numerics

Scheme:

- large literature on numerical routines for hyperbolic systems of PDEs.
- Rhebergen et al. (2008) developed a novel discontinuous Galerkin (DG) finite element framework for hyperbolic system of PDEs with non-conservative products $\mathsf{G}(\mathsf{U})\partial_x \mathsf{U}$.
- in most simple case (DG0), analagous to Godunov's FV scheme in which a numerical flux must be evaluated

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathsf{U}_k + \frac{1}{\triangle x_k}\left[P^{NC}(\mathsf{U}_k, \mathsf{U}_{k+1}) - P^{NC}(\mathsf{U}_{k-1}, \mathsf{U}_k)\right] + \frac{\mathsf{S}(\mathsf{U}_\mathsf{k})}{\triangle x_k} = 0.$$

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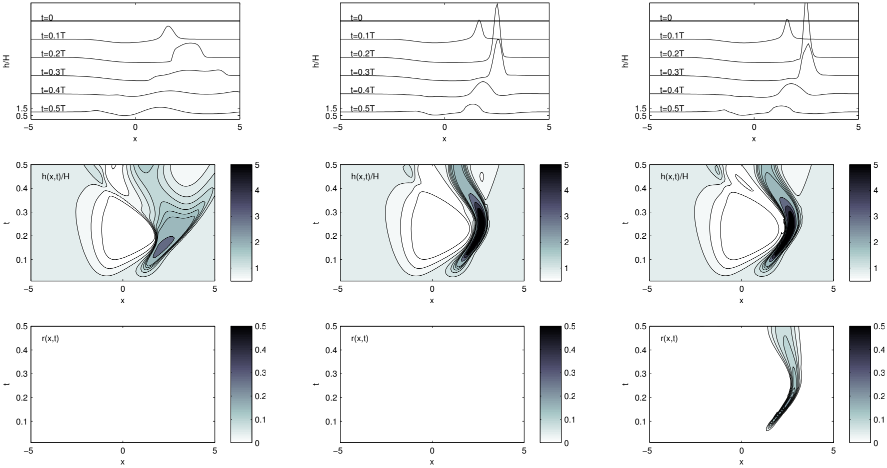
Experiments:

- Rossby geostrophic adjustment in a periodic domain
- describes the evolution of the free surface height h when disturbed from its rest state by a transverse jet, i.e., fluid with an initial constant height profile is subject to a localised v-velocity distribution.
- non-dimensional parameters: $\mathrm{Ro} = 1$ and $\mathrm{Fr} = 2$.

Adjustment of a transverse jet in RSW

Below H_c and H_r: | Above H_c but below H_r: | Above H_c and H_r:



2. Ensemble-based DA for idealised models

<u>Ensemble Kalman filter</u>: twin model setting

- Imperfect model:
  - "truth" trajectory: run at high resolution
  - "forecast" model: run at lower resolution at which small-scale features (e.g., localised moisture transport) are not fully resolved
  - ensemble (covariance) inflation $(\mathbf{x}_i^f \leftarrow \gamma(\mathbf{x}_i^f - \overline{\mathbf{x}}^f) + \overline{\mathbf{x}}^f)$ applied to account for the model error due to resolution mismatch
  - localisation $(\mathbf{P}^f \leftarrow \rho_{loc} \circ \mathbf{P}^f)$ applied to damp spurious long-range correlations

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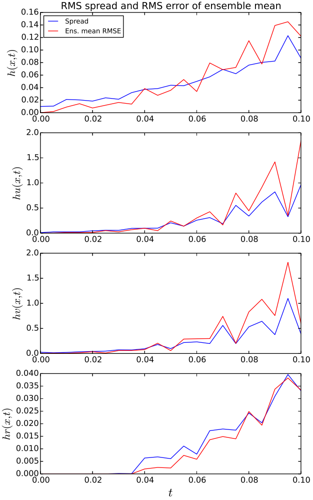
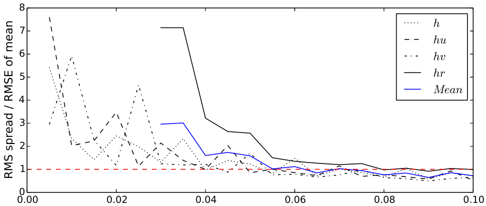
  - localisation ($\mathbf{P}^f \leftarrow \rho_{loc} \circ \mathbf{P}^f$) applied to damp spurious long-range correlations
- "tuning" the observing system: what to observe? how often? with how much noise?
- observational influence diagnostic (after Cardinali et al. (2004)) averaged over cycles:

$$OI = \frac{tr(\mathbf{HK})}{p}$$

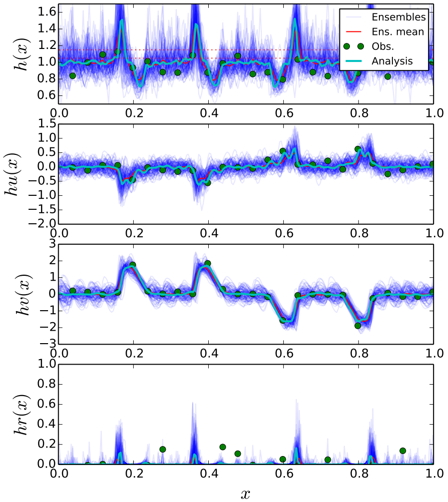
Before assimilating...: ensemble spread as a representation of forecast error



Ratio of ensemble spread ($N = 100$) to forecast error:



Cycled assimilation...: how does an analysis look?



Field-averaged RMS errors after an analysis cycle (Obs. error = 0.1):

	Forecast	Analysis
h	0.0731	**0.0725**
hu	0.1052	**0.0812**
hv	0.1374	**0.0696**
hr	**0.0169**	0.0238

Observational influence diagnostic:

$$OI = \frac{tr(\mathbf{HK})}{p} = 0.28$$

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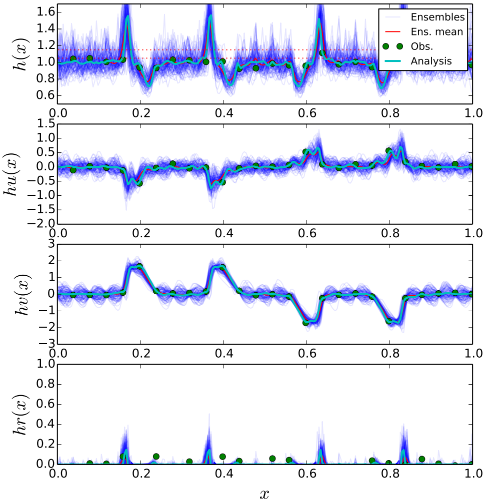

Field-averaged RMS errors after an analysis cycle (Obs. error = 0.05):

	Forecast	Analysis
h	0.0828	**0.0816**
hu	0.0991	**0.0906**
hv	0.1297	**0.0793**
hr	**0.0200**	0.0293

Observational influence diagnostic:

$$OI = \frac{tr(\mathbf{HK})}{p} = 0.42$$

Lots of parameters and different set-ups to explore and play with:

- observe only one variable (e.g., the height field) and compare; or observe nonlinearly (e.g., radial wind)
- include topography and observe downstream of a mountain
- increase the ratio of truth to forecast resolution to observe smaller-scale features
- (too) many more possibilities...

3. Current/future work and ideas

DA:

- setting up a demonstration system that compares EnKF with VAR in which B matrix is derived from ensemble.

Numerics:

- extension to ensure non-negativity of hr, à la Audusse et al., 2004.

$$P^{NC}(\mathsf{U}_k, \mathsf{U}_{k+1}) \longrightarrow P^{NC}(\mathsf{U}_{(k+1/2)-}, \mathsf{U}_{(k+1/2)+})$$

- reconstructed states $\mathsf{U}_{(k+1/2)\pm}$ impose that h and hr cannot become negative yet dry states $hr = 0$ can be computed (given a derived time-step criterion).

Other models of interest:

- (dimensionally-reduced) adapted moist Boussinesq shallow water equations (after Zerroukat and Allen, 2015)
- 3D QG model with anisotropic rotating convection (Bokhove et al., poster)

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Other diagnostics and the question of 'relevance':

- how can findings based on 'toy' models generalise to and provide useful insight for operational NWP forecast/assimilation systems?
- observational influence diagnostic:
  - global NWP: 0.15 (Cardinali et al., 2004)
  - convective-scale NWP: 0.2 - 0.5? (Brousseau et al., 2014)

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- observational influence diagnostic:
  - global NWP: 0.15 (Cardinali et al., 2004)
  - convective-scale NWP: 0.2 - 0.5? (Brousseau et al., 2014)
- error-growth properties of the idealised model should be similar to those in operational models:
  - error-growth characteristics of assimilating model determine magnitude and structure of the updated $\mathbf{P}^f$ represented by the ensemble.
  - error-doubling time for forecast error for global NWP known to be on the order of days - what about convective-scale?

Summary and outlook

- novel fluid dynamical models to fill the ‘complexity gap’ between ODE models and the primitive equations / state-of-the-art NWP models
- Idealised convective-scale DA experiments with characteristics relevant for NWP
- Implement a variational algorithm (in which initial covariance comes from the ensemble)
- Integrate model(s) into Met Office’s nascent ‘VarPy’ framework as a repository for idealised DA experiments

Thank you very much for your attention.

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