# Multi-Scale Data Assimilation for Fine-Resolution Models

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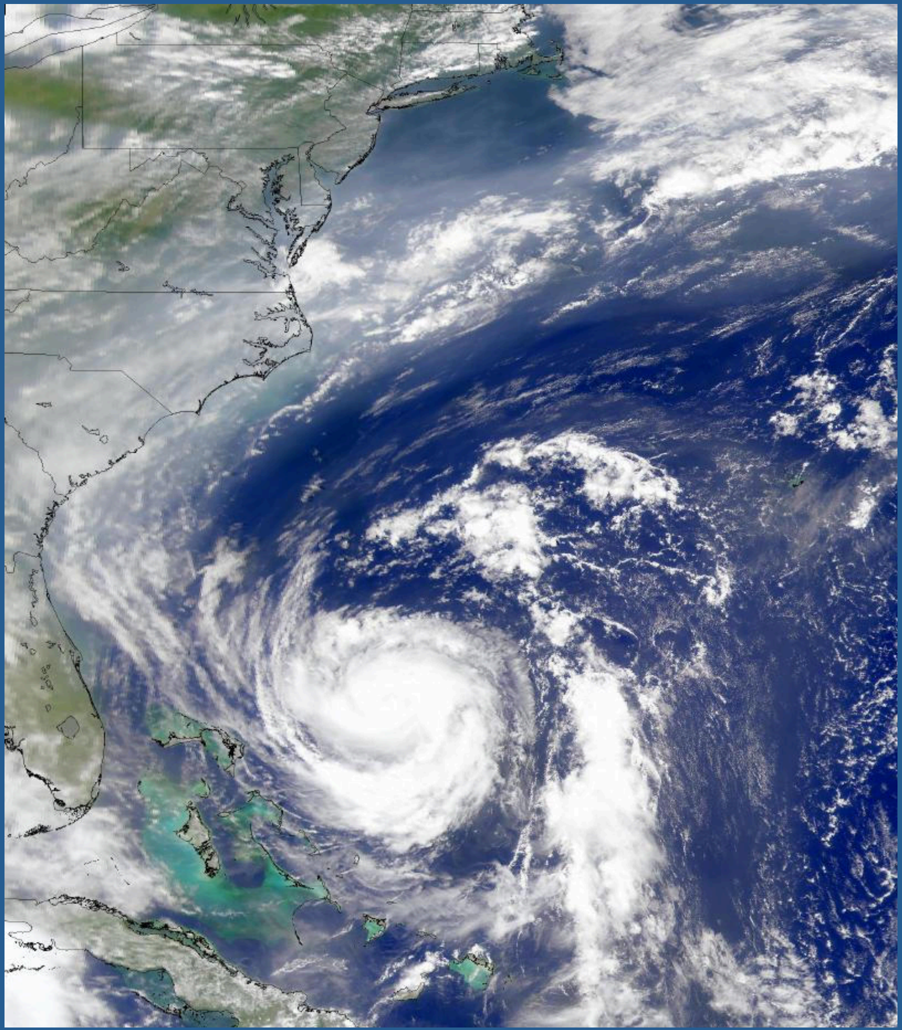
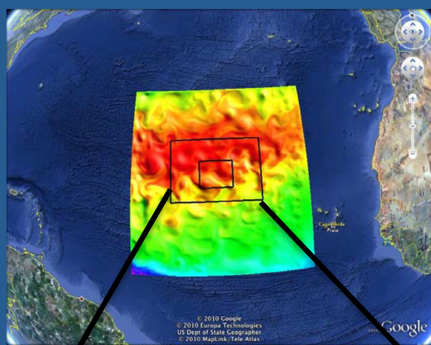
# Atmospheric Data Assimilation for Cloud Resolving Models

- Regional operational models often have a resolution of higher than 4 km to resolve cloud systems
- Are data assimilation schemes based on optimal estimation theory are suitable for cloud resolving models?

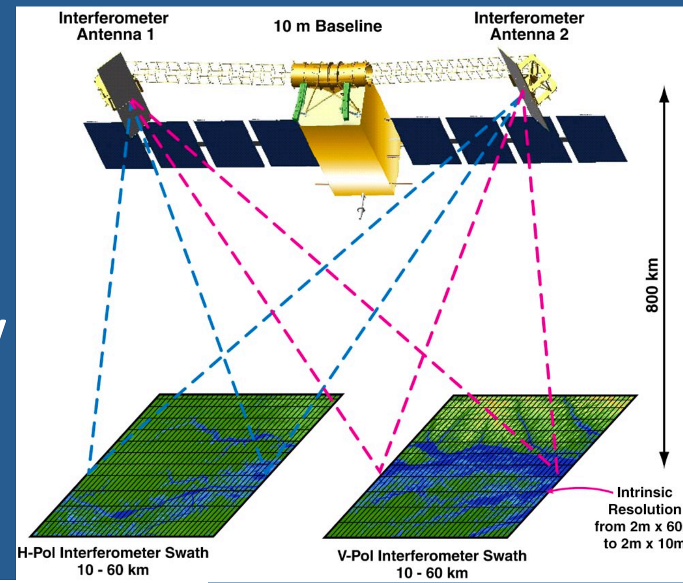


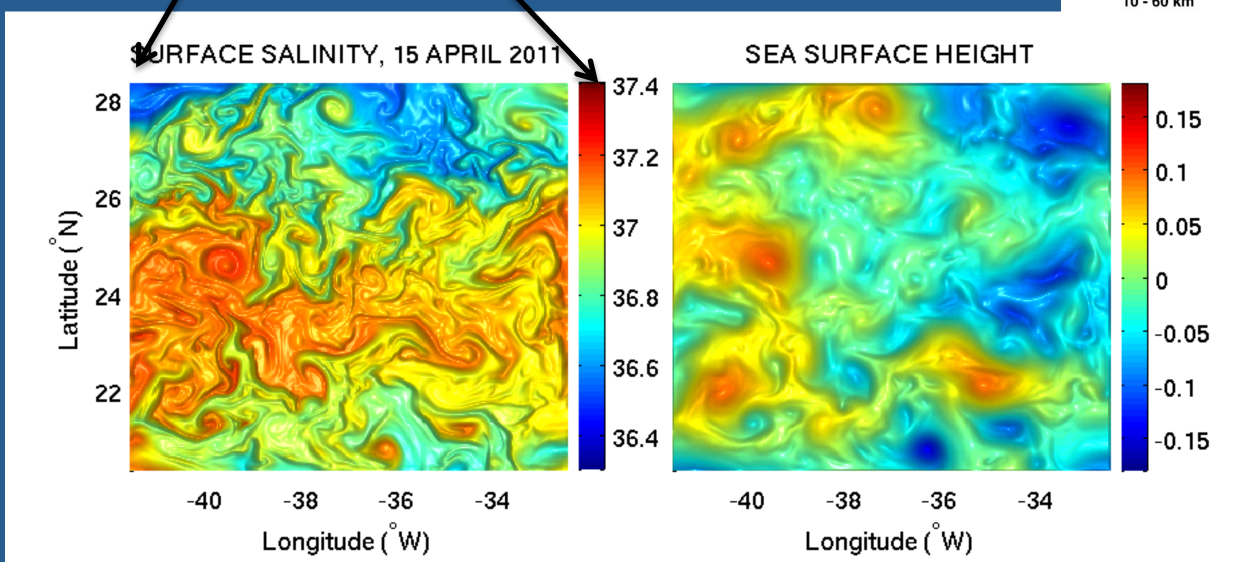
# Oceanic Data Assimilation for Sub-Mesoscale Processes

- Mesoscale 40km – 400 km
- Sub-mesoscale 1 km - 40 km



Surface Water and Ocean Topography (SWOT) Satellite




Salinity Processes in the Upper Ocean Regional Study (SPURS) (http://spurs.jpl.nasa.gov)

# Conventional Data Assimilation: Optimal Estimation

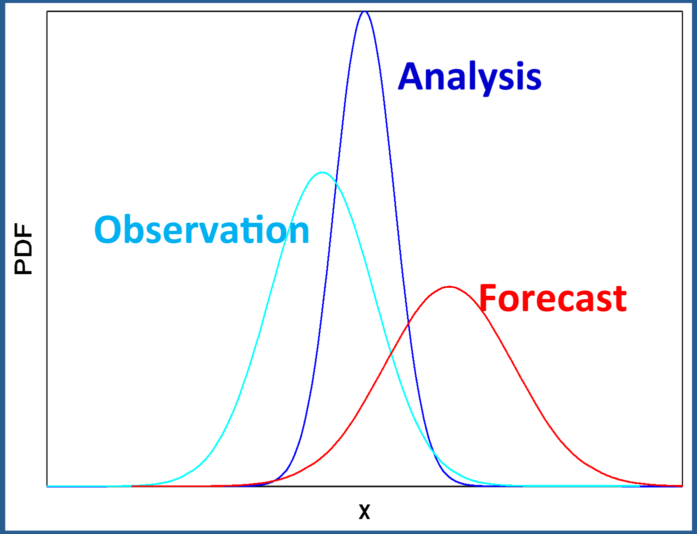
$$\min_x J = \frac{1}{2}(x - x^b)^T B^{-1}(x - x^b) + \frac{1}{2}(Hx - y)^T R^{-1}(Hx - y)$$

Variational methods (3Dvar/4Dvar):

- prescribed B
- optimization algorithm

Sequential methods (Kalman filter/smoother)

- dynamically evolved B
- analytical solution (matrix manipulations)



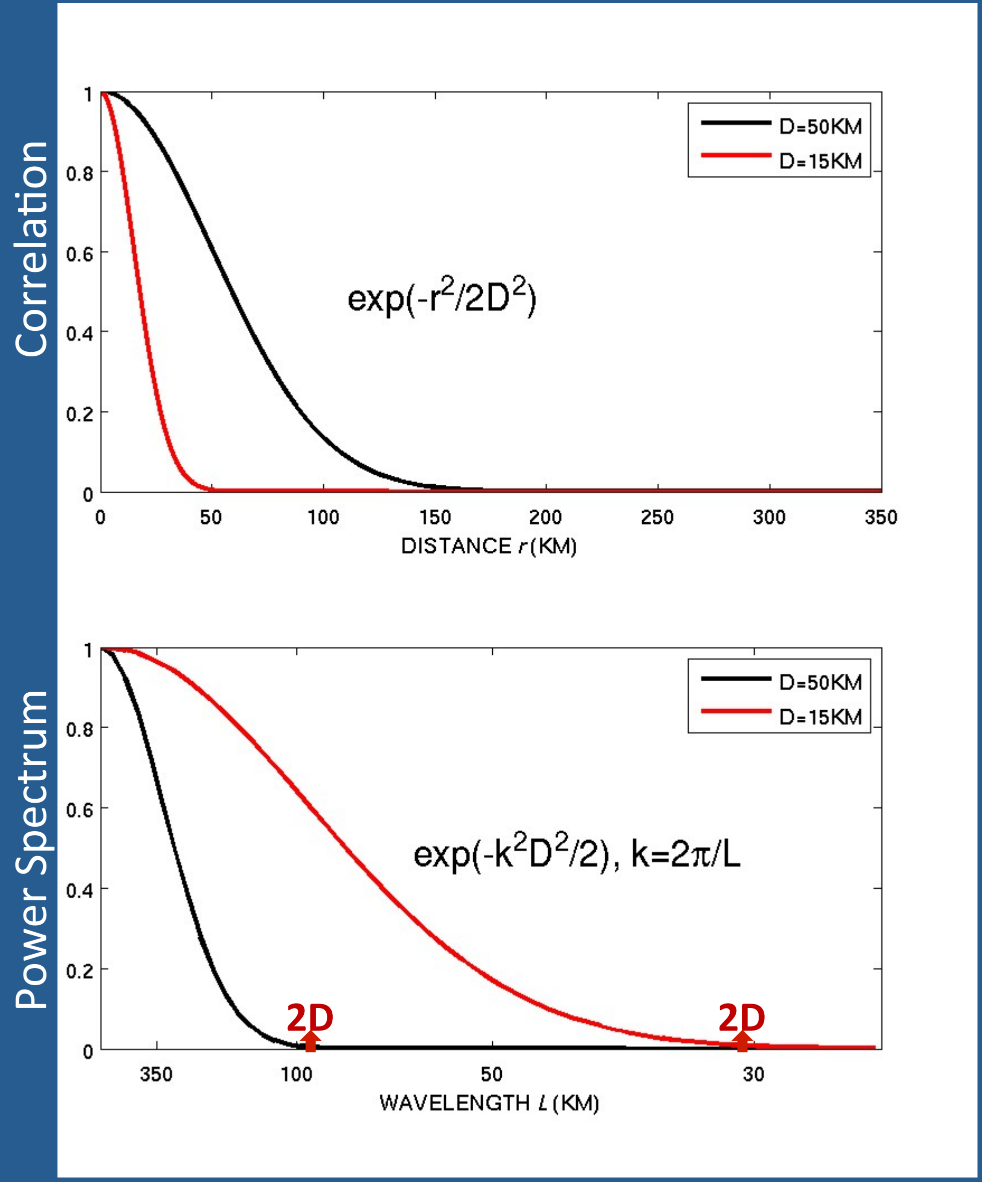

Maximum Likelihood

# Error Covariance: Spreading and Filtering

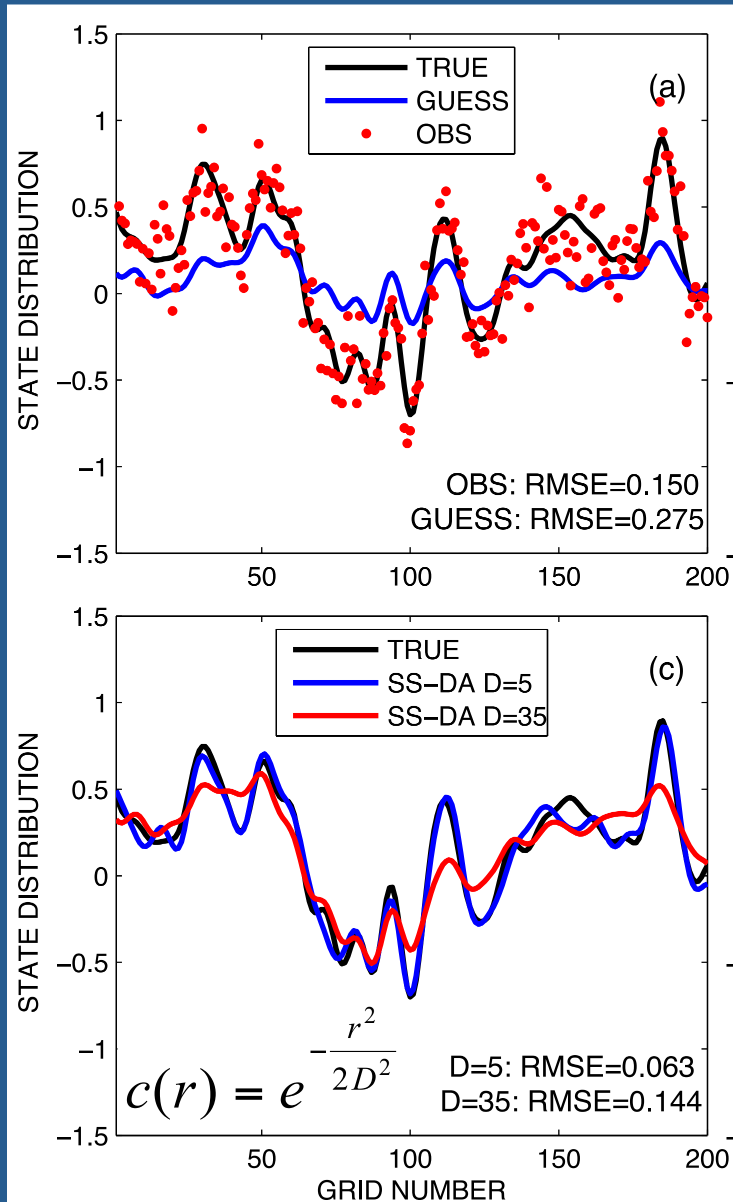
$$B = \Sigma C \Sigma$$

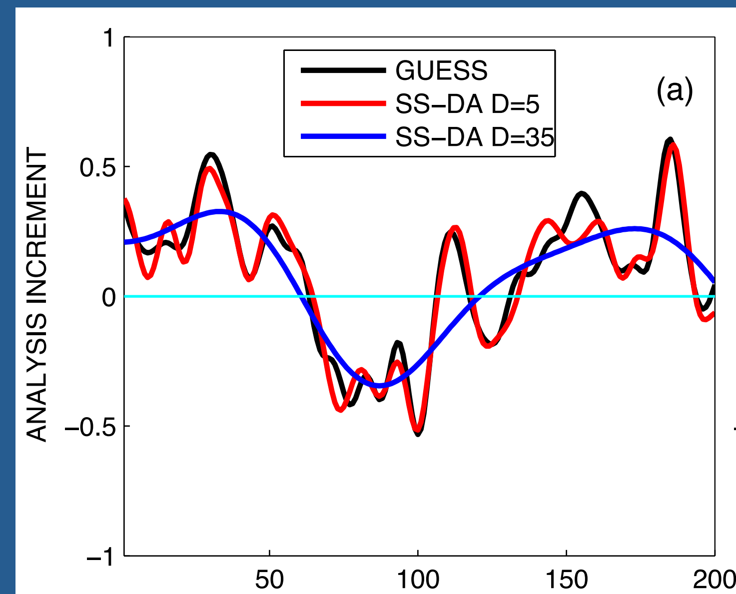
$$c(r) = e^{-\frac{r^2}{2D^2}}$$

$$c_n = \frac{D}{\sqrt{2\pi}} \exp\left(-\frac{k^2 D^2}{2}\right)$$



# Filtering Properties
# An Idealized 1D Experiment




**With a large correlation scale, DA corrects the large scale error**

# 1D Problem with Homogeneous and Isotropic Background Error

For both global and regional domains, we have

$$B_S \approx FBF^T = \sigma^{b2} S$$

$$S = FCF^T$$

**F** – Discrete Fourier Transform (DFT)

- The eigenvalues of ***C*** are its Fourier transform coefficients, namely, values of the spectral density function.
- The vectors that define the discrete Fourier transform are eigenvectors of ***C***.

# Filtering Properties

$$x^a = x^b + BH^T\left(HBH^T + R\right)^{-1}\left(y - Hx^b\right)$$

$$H = I$$

$$s^a = Fx^a$$

$$s^d = F\left(y - x^f\right)$$

$$B_S = FBF^T = \sigma^{b2} S$$

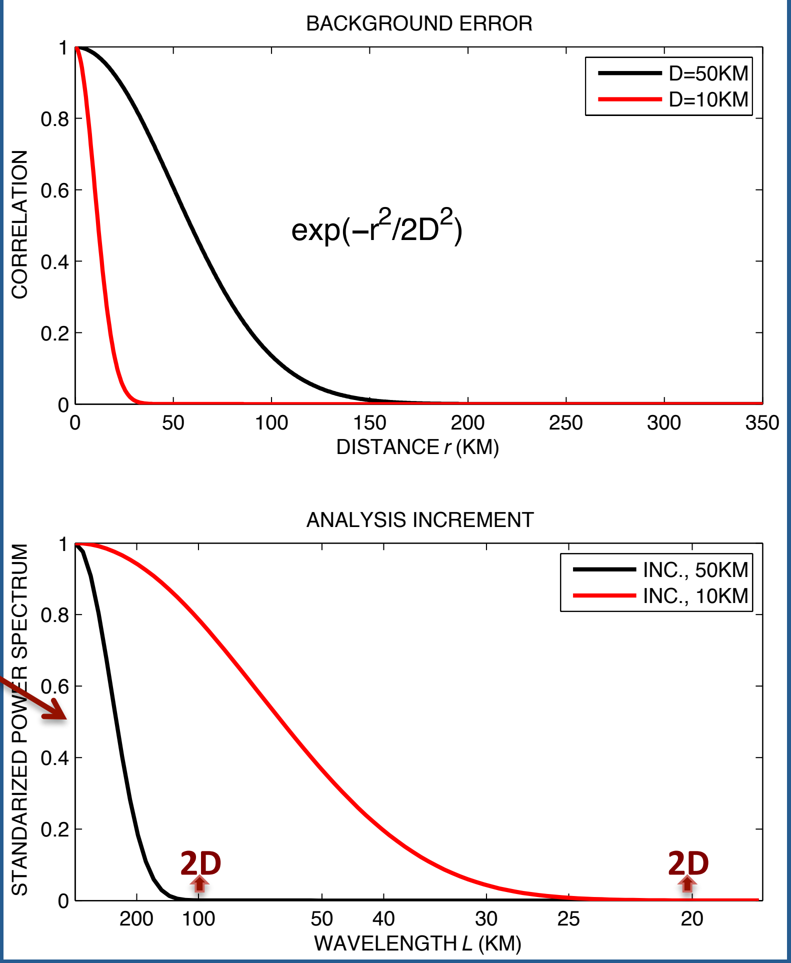
$$R_S = FRF^T = \sigma^{o2} I$$

$$s^a = s^b + S\left(S + \frac{\sigma^{o2}}{\sigma^{b2}} I\right)^{-1} s^d$$

# Filtering Properties: Analysis Increment Scales

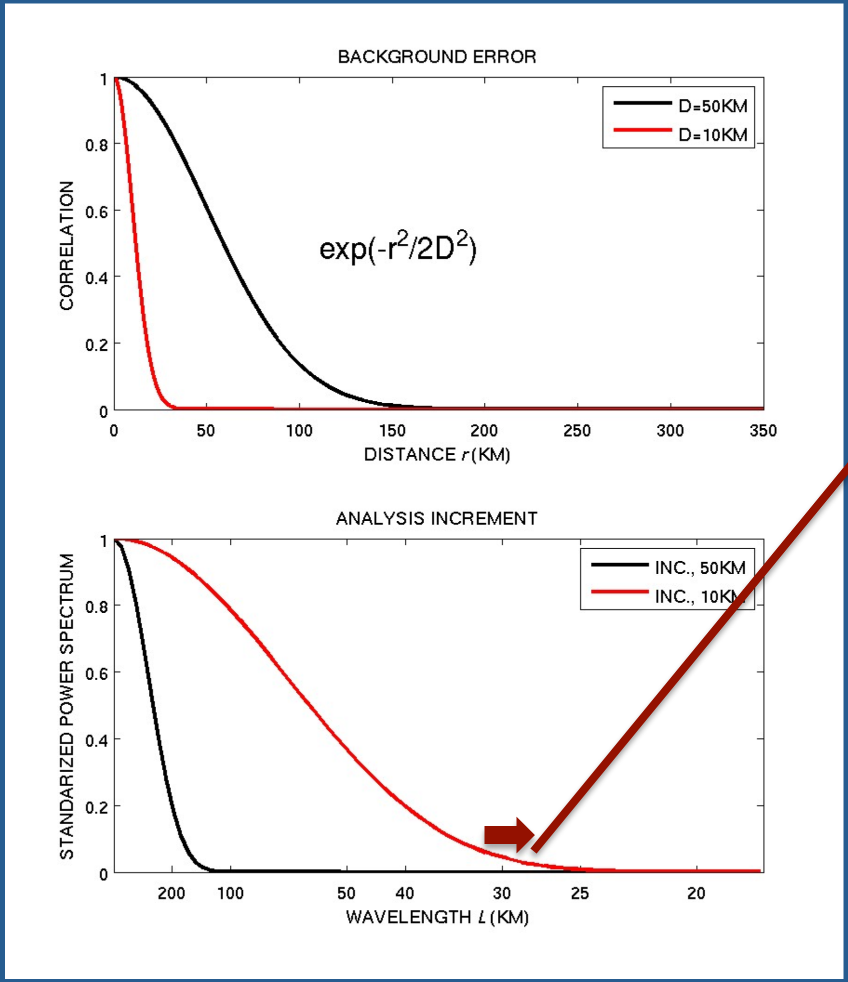
$$s^a = s^b + S\left(S + \frac{\sigma^{o2}}{\sigma^{b2}} I\right)^{-1} s^d$$

$$\frac{\sigma^o}{\sigma^b} = \frac{1}{2}$$




**Conventional data assimilation is unable to update fine-scale information**

# Meso- and Small Scale Component in Background Error Covariance




1. Meso- and small- scale systems are intensive, but are localized and intermittently occur.
2. The forecast/background error covariance is primarily determined by large scale systems
3. The correlation scale is inevitable to be large scale

# Questions and Challenges

- Do we need to update fine-scale information in data assimilation?

  maybe not ?

- If yes, how can we update fine scale information?

**We here suggest a scheme to update fine scale information**

# Multi-Scale Data Assimilation: Data Assimilation Separately for Distinct Scales

**Scale decomposition**

$$x = x_L + x_S$$
$$e = e_L + e_S$$

$$\left\langle e_L e_S^T \right\rangle = 0$$
$$B = B_L + B_S$$

**Multi-scale DA**

$$\min_{\delta x_L} J = \frac{1}{2}\delta x_L{}^T B_L^{-1}\delta x_L + \frac{1}{2}(H\delta x_L - \delta y)^T \left(HB_S H^T + R\right)^{-1}(H\delta x_L - \delta y)$$
$$\min_{\delta x_S} J = \frac{1}{2}\delta x_S{}^T B_S^{-1}\delta x_S + \frac{1}{2}(H\delta x_S - \delta y)^T \left(HB_L H^T + R\right)^{-1}(H\delta x_S - \delta y)$$

$$\delta x = x - x^b$$

(Li et al., 2015, MWR)

# Properties of Multi-Scale Data Assimilation

- The decomposed cost function can be derived by maximizing the conditional probability

$$p(x_L \mid y)$$

$$p(x_S \mid y)$$

- Separate estimate of the state for distinct scales using decomposed cost functions
- Explicit incorporation of multiple decorrelation scales, thus, **Multi-Scale Data Assimilation**

# Multi-Scale Representativeness Errors and Aliasing

**Scales untangled**

$$s^a = s^b + S\left(S + \frac{\sigma^{o2}}{\sigma^{b2}} I\right)^{-1} s^d$$

**Scales tangled** → **Scale aliasing/contamination ?**

$$\min_{\delta x_L} J = \frac{1}{2}\delta x_L{}^T B_L^{-1}\delta x_L + \frac{1}{2}(H\delta x_L - \delta y)^T \left(\underline{HB_S H^T} + R\right)^{-1}(H\delta x_L - \delta y)$$

$$\min_{\delta x_S} J = \frac{1}{2}\delta x_S{}^T B_S^{-1}\delta x_S + \frac{1}{2}(H\delta x_S - \delta y)^T \left(\underline{HB_L H^T} + R\right)^{-1}(H\delta x_S - \delta y)$$

**Multi-scale representativeness error**

# Assimilation of Decomposed Observations

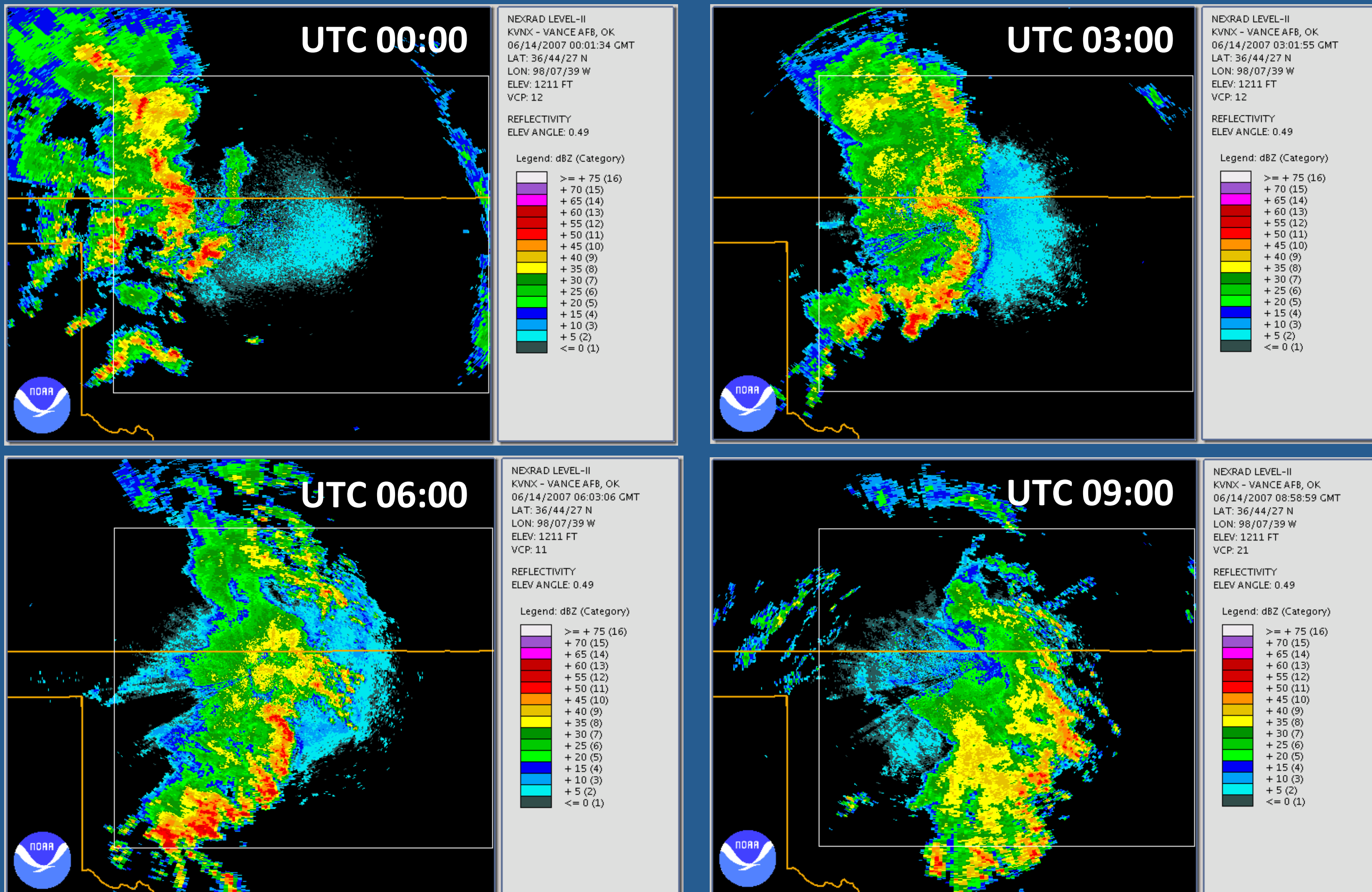
$$\delta y = \delta y_L + \delta y_S$$

$$\min_{\delta x_L} J = \frac{1}{2}\delta x_L^T B_L^{-1}\delta x_L + \frac{1}{2}(H\delta x_L - \delta y_L)^T \left(\cancel{HB_S H^T} + R_L\right)^{-1}(H\delta x_L - \delta y_L)$$

$$\min_{\delta x_S} J = \frac{1}{2}\delta x_S^T B_S^{-1}\delta x_S + \frac{1}{2}(H\delta x_S - \delta y_S)^T \left(\cancel{HB_L H^T} + R_S\right)^{-1}(H\delta x_S - \delta y_S)$$

**Multi-scale representativeness error**

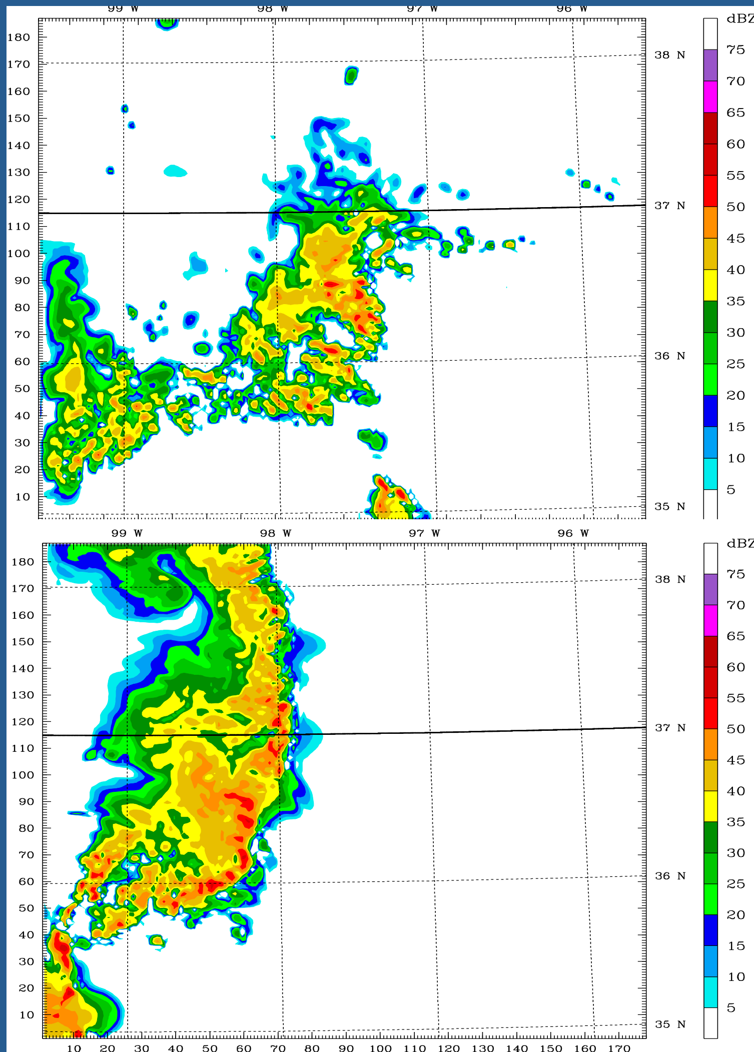
# NEXRAD Reflectivity: Meso-Scale Connective System (MCS): June 14, 2007



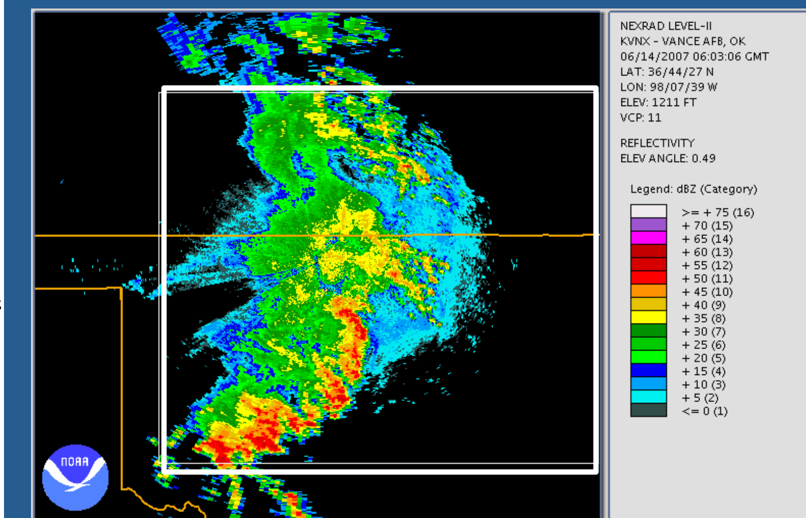
# Improved Reflectivity: UTC 06, 14 June, 2007

NO DA

DA




Simulated 900 hPa Reflectivity




(Li et al., 2015, JGR)

# Summary

- Due to the filtering properties, conventional data assimilation can not effectively update  fine-scale information.
- To update fine-scale information, it is suggested that the fine scale should be treated separately from larger scales.
- The cost function is mathematically decomposed for formulating a MS-DA scheme .
- The decomposed cost function allows for the background error covariance to explicitly incorporate multiple decorrelation scales.
- Experiments show promising performance of the MS-DA scheme in 3Dvar for both oceanic and atmospheric applications