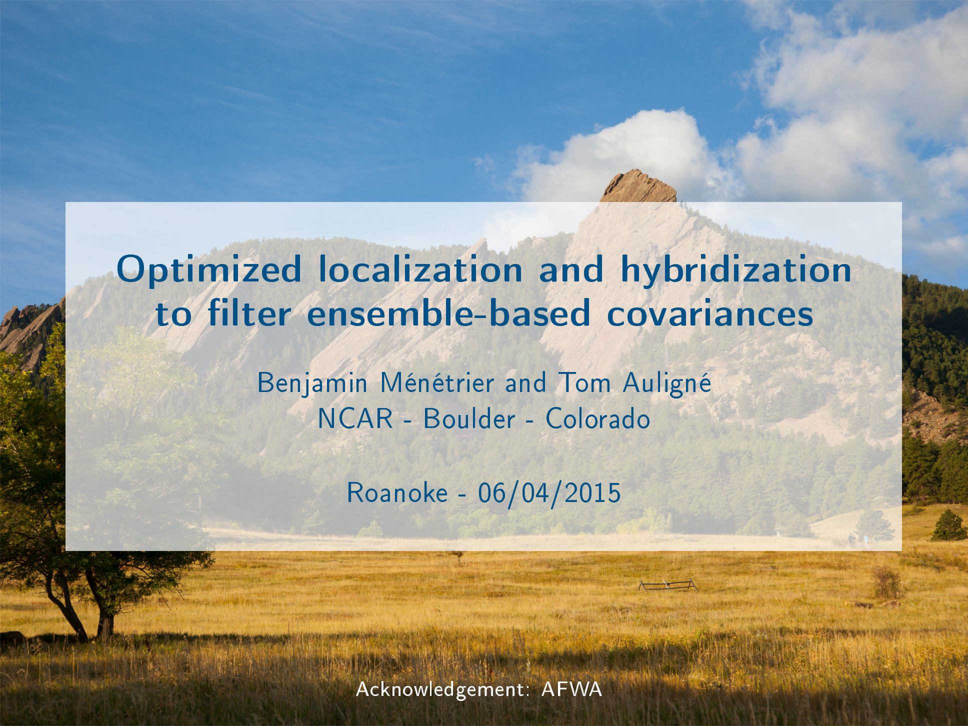

Optimized localization and hybridization to filter ensemble-based covariances

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NCAR - Boulder - Colorado

Roanoke - 06/04/2015

Acknowledgement: AFWA

Introduction

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- Covariance hybridization
  $\rightarrow$ linear combination with a static covariance matrix

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   - use data from the **ensemble only**.
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3. Is hybridization **always** improving the accuracy of forecast error covariances?

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Theory of sampling error:

$$\mathbb{E}\left[\widetilde{B}_{ij}^2\right] = \frac{N(N-3)}{(N-1)^2}\mathbb{E}\left[\widetilde{B}_{ij}^{\star 2}\right] - \frac{1}{(N-1)(N-2)}\mathbb{E}\left[\widetilde{B}_{ii}\widetilde{B}_{jj}\right] + \frac{N^2}{(N-1)^2(N-2)}\mathbb{E}\left[\widetilde{\Xi}_{ijij}\right]$$

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$\boxed{\text{Localization by } \mathbf{L} \text{ (Schur product)}}$

Covariance matrix

$$\widehat{\mathbf{B}} = \mathbf{L} \circ \widetilde{\mathbf{B}}$$

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Increment

$$\delta \mathbf{x}^e = \frac{1}{\sqrt{N-1}} \sum_{p=1}^{N} \delta \widetilde{\mathbf{x}}_p^b \circ \left(\mathbf{L}^{1/2} \mathbf{v}_p^\alpha \right)$$

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$\boxed{\text{Localization by } \mathbf{L} + \text{hybridization with } \overline{\mathbf{B}}}$

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$$\delta \mathbf{x} = \beta^e \; \delta \mathbf{x}^e + \beta^c \; \overline{\mathbf{B}}^{1/2} \mathbf{v}^c$$

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Localization + hybridization = linear filtering of $\widetilde{\mathbf{B}}$

$\mathbf{L}^h$ and β^c have to be optimized together

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Goal: to minimize the expected quadratic error:

$$e = \mathbb{E}\left[\| \underbrace{\mathbf{L} \circ \widetilde{\mathbf{B}}}_{\text{Localized } \widetilde{\mathbf{B}}} - \underbrace{\widetilde{\mathbf{B}}^{\star}}_{\text{Asymptotic } \widetilde{\mathbf{B}}} \|^2 \right] \tag{1}$$

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An **explicit formula** for the optimal localization $\mathbf{L}$ is given in Ménétrier et al. 2015 (Montly Weather Review).

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This formula of optimal localization $\mathbf{L}$ involves:

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$$L_{ij} = \frac{(N-1)^2}{N(N-3)} - \frac{N}{(N-2)(N-3)} \frac{\mathbb{E}\left[\widetilde{\Xi}_{ijij}\right]}{\mathbb{E}\left[\widetilde{B}_{ij}^2\right]} + \frac{N-1}{N(N-2)(N-3)} \frac{\mathbb{E}\left[\widetilde{B}_{ii}\widetilde{B}_{jj}\right]}{\mathbb{E}\left[\widetilde{B}_{ij}^2\right]}$$

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Step 2: optimizing localization and hybridization **together**

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$$e^h = \mathbb{E}\left[\| \underbrace{\mathbf{L}^h \circ \widetilde{\mathbf{B}} + (\beta^c)^2 \overline{\mathbf{B}}}_{\text{Localized / hybridized } \widetilde{\mathbf{B}}} - \underbrace{\widetilde{\mathbf{B}}^\star}_{\text{Asymptotic } \widetilde{\mathbf{B}}} \|^2\right]$$

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Result of the minimization: a linear system in $\mathbf{L}^h$ and $(\beta^c)^2$

$$L^h_{ij} = L_{ij} - \frac{\mathbb{E}\left[\widetilde{B}_{ij}\right]}{\mathbb{E}\left[\widetilde{B}^2_{ij}\right]} \overline{B}_{ij} \; (\beta^c)^2 \tag{2a}$$

$$(\beta^c)^2 = \frac{\sum_{ij} \overline{B}_{ij} (1 - L^h_{ij}) \mathbb{E}\left[\widetilde{B}_{ij}\right]}{\sum_{ij} \overline{B}^2_{ij}} \tag{2b}$$

Hybridization benefits

Comparison of:

- $\widehat{\mathbf{B}} = \mathbf{L} \circ \widetilde{\mathbf{B}}$, with an optimal $\mathbf{L}$ minimizing e
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We can show that:

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With optimal parameters, whatever the static $\overline{\mathbf{B}}$:
Localization + hybridization is better than localization alone

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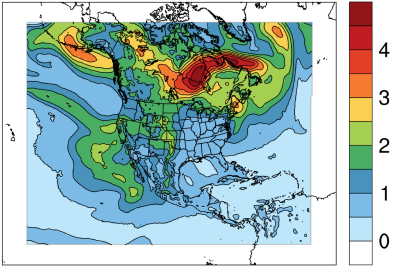
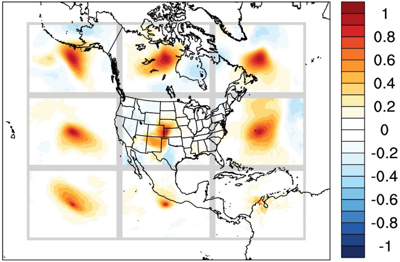
Temperature at level 7 ($\sim$ 1 km above ground), 48 h-range forecasts



Standard-deviation (K)



Correlations functions

Localization and hybridization

- Optimization of the horizontal localization $\mathbf{L}^h_{\mathrm{hor}}$ and of the hybridization coefficient β^c at each vertical level.

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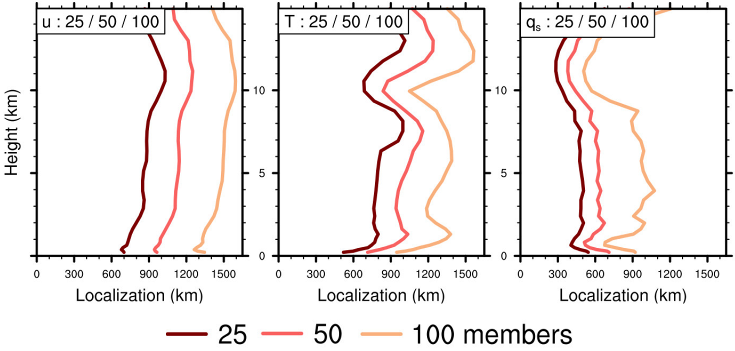

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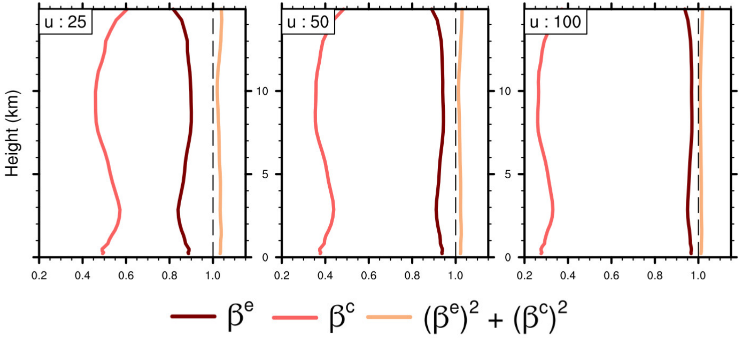
- Hybridization coefficients for zonal wind:



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4.5 %	4.2 %	3.9 %	1.7 %

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Ménétrier, B. and T. Auligné: Optimized Localization and Hybridization to Filter Ensemble-Based Covariances
Monthly Weather Review, **2015**, accepted

Perspectives

Already done in the paper:

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- Extension to vectorial hybridization weights:

$$\delta \mathbf{x} = \boldsymbol{\beta}^e \circ \delta \mathbf{x}^e + \boldsymbol{\beta}^c \circ \delta \mathbf{x}^c$$

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$\rightarrow$ Requires the solution of a nonlinear system $\mathcal{A}(\mathbf{L}^h, \boldsymbol{\beta}^c) = 0$, performed by a bound-constrained minimization.

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To be done:

- Tests in a cycled quasi-operational configuration
- Extension of the theory to account for systematic errors in $\widetilde{\mathbf{B}}^\star$ (theory is ready, tests are underway...)

Thank you for your attention!

Any question?