Addressing the low-order clustering in deterministic filters due to nonlinear dynamics using mean-preserving non-symmetric solutions of the ETKF

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Abstract

We propose a **modified Local Ensemble Transform Kalman Filter (LETKF)** that **avoids ensemble clustering without deteriorating balance.** A **constrained resampling** is achieved by **mean-preserving random rotations** of the ensemble perturbations**.**

1. Low order clustering in EnSRFs as a result of nonlinearity

Kalman filtering is **optimal** when the **forecast model** is **linear** and the **model error** and **observational error** follow **Gaussian** statistics. Usually, these conditions are not perfectly fulfilled. How well they are **approximated** depends upon the **length of the assimilation window** and the **magnitude** of the **model and observational error covariance**.

Previous works (Lawson and Hansen, 2004; Anderson, 2010) studied the behavior of the **stochastic ensemble Kalman filter** (EnKF Burgers *et al*, 1998) and [deterministic] **ensemble square root filters** (EnSRF, Tippett *et al* 2003) when **nonlinear effects become important**. A **deformation of the M-member ensemble** was observed in the case of **EnSRFs**; namely, **an ensemble member becomes an outlier while the rest of the members collapse in a cluster** (to preserve the variance), affecting the performance and the higher order moments of the ensemble. The **stochastic EnKF** doesn't present this problem, but **additional sampling noise is introduced** due to the random number realizations, especially in small ensembles.

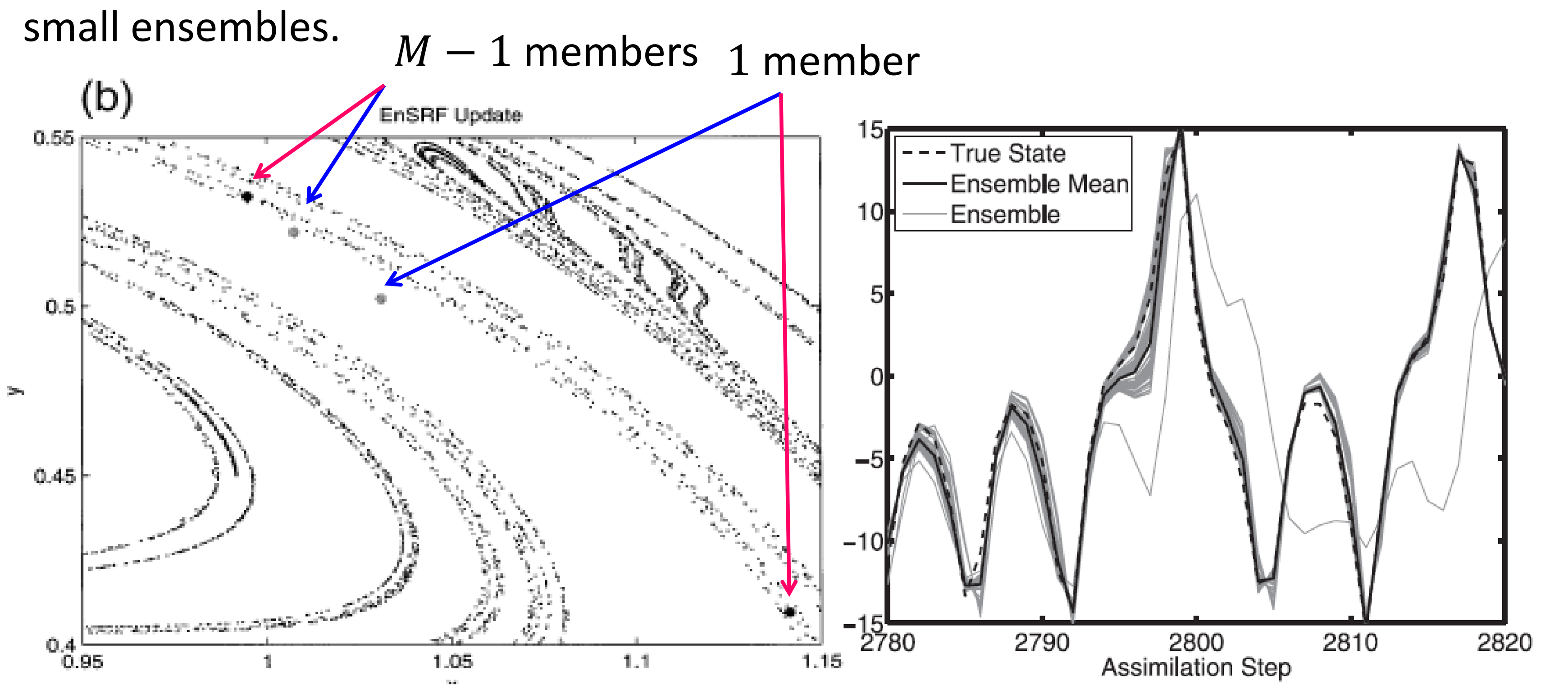

Figure 1. Left: experiment with the Ikeda system and the **serial EnSRF** showing the **low order clustering for both background and analysis ensembles** at a single time, taken from Lawson and Hansen 2004. Right: experiment with the Lorenz 1963 model and the **Ensemble Adjustment Kalman Filter** (EAKF) showing the **ensemble clustering in the time evolution of the analysis ensemble** for one variable, taken from Anderson 2010.

2. A Mean-Preserving Non-Symmetric Ensemble Transform Kalman Filter (MPNS-ETKF)

The **ETKF** is a member of the **EnSRF family**. In this scheme, the **analysis ensemble of perturbations** $\mathbf{X}^a \in \Re^{N\times M}$ is obtained by **post-multiplying** the **background ensemble of perturbations** $\mathbf{X}^b \in \Re^{N\times M}$ by a matrix of weights $\mathbf{W}^a \in \Re^{M\times M}$. The resulting $\mathbf{X}^a = \mathbf{X}^b\mathbf{W}^a$ must fulfill two requisites:

a) It must **respect the Kalman filter covariance equation** $\mathbf{P}^a = (\mathbf{I} - \mathbf{KH})\mathbf{P}^b$
b) It must **have mean zero**, i.e. $\mathbf{X}^a\mathbf{1} = \mathbf{0}$

The original (**one-sided**) **ETKF** (Bishop *et al*, 2001) is based in the **singular value decomposition of the multidimensional ratio of background and observational error covariance** to form the transform matrix:

$$\mathbf{W}^a_{1-sided} = \mathbf{C}(\mathbf{I}+\mathbf{\Gamma})^{-\frac{1}{2}} \qquad \mathbf{C\Gamma C}^{\mathrm{T}} = \frac{\mathbf{Y}^{b\,\mathrm{T}}\mathbf{R}^{-1}\mathbf{Y}^b}{M-1}$$

The columns of $\mathbf{C}$ are eigenvectors and $\mathbf{\Gamma}$ contains eigenvalues in the diagonal. Although it clearly respects the covariance equation, this formulation doesn't preserve the zero mean (the original purpose was adaptive sampling rather than data assimilation). A mean-preserving symmetric solution was proposed by Wang *et al* 2004 (**spherical simplex**) and Hunt *et al* 2007 (**L[ocal]ETKF**); it yields the **transform matrix closest to the identity** (Ott *et al* 2004):

$$\mathbf{W}^a_{LETKF} = \mathbf{C}(\mathbf{I}+\mathbf{\Gamma})^{-\frac{1}{2}}\mathbf{C}^{\mathrm{T}}$$

A **general mean-preserving non-symmetric solution for the ETKF** can be written as:

$$\mathbf{W}^a_{MPNS-ETKF} = \mathbf{C}(\mathbf{I}+\mathbf{\Gamma})^{-\frac{1}{2}}\mathbf{S}^{\mathrm{T}}$$

$\mathbf{S} \in \Re^{M\times M}$ must be **orthonormal** $\mathbf{S}^{\mathrm{T}}\mathbf{S} = \mathbf{I}$ and be such that $\mathbf{W}^a_{MPNS-ETKF}$ **contains 1 as an eigenvector**. Simple and cheap $O(M^3)$ forms to construct this matrix are available (Bishop, *pers comm*).

The **MPNS-ETKF** analysis at each assimilation instant can be considered a **random rotation of the one-sided ETKF**. Its effect can be viewed **as a constrained resampling of the ensemble** (which doesn't add external noise as the stochastic EnKF).

3. Using the MPNS-ETKF to avoid low-order clustering

We illustrate the effects of the **MPNS-ETKF** using **a simple univariate quadratic model**:

$$\dot{x} = x + bx^2 \quad \rightarrow \quad x_{t+1} = (1+\Delta)x_t + b\Delta {x_t}^2$$

$\Delta = 0.05$ is the Euler time step of the discretized **model** and b controls the degree of non-linearity. The unstable fixed point $x^* = 0$ is set as the truth. For the set of experiments shown here, the observational error variance R and the initial **background** variance P_0^b were set to 1.

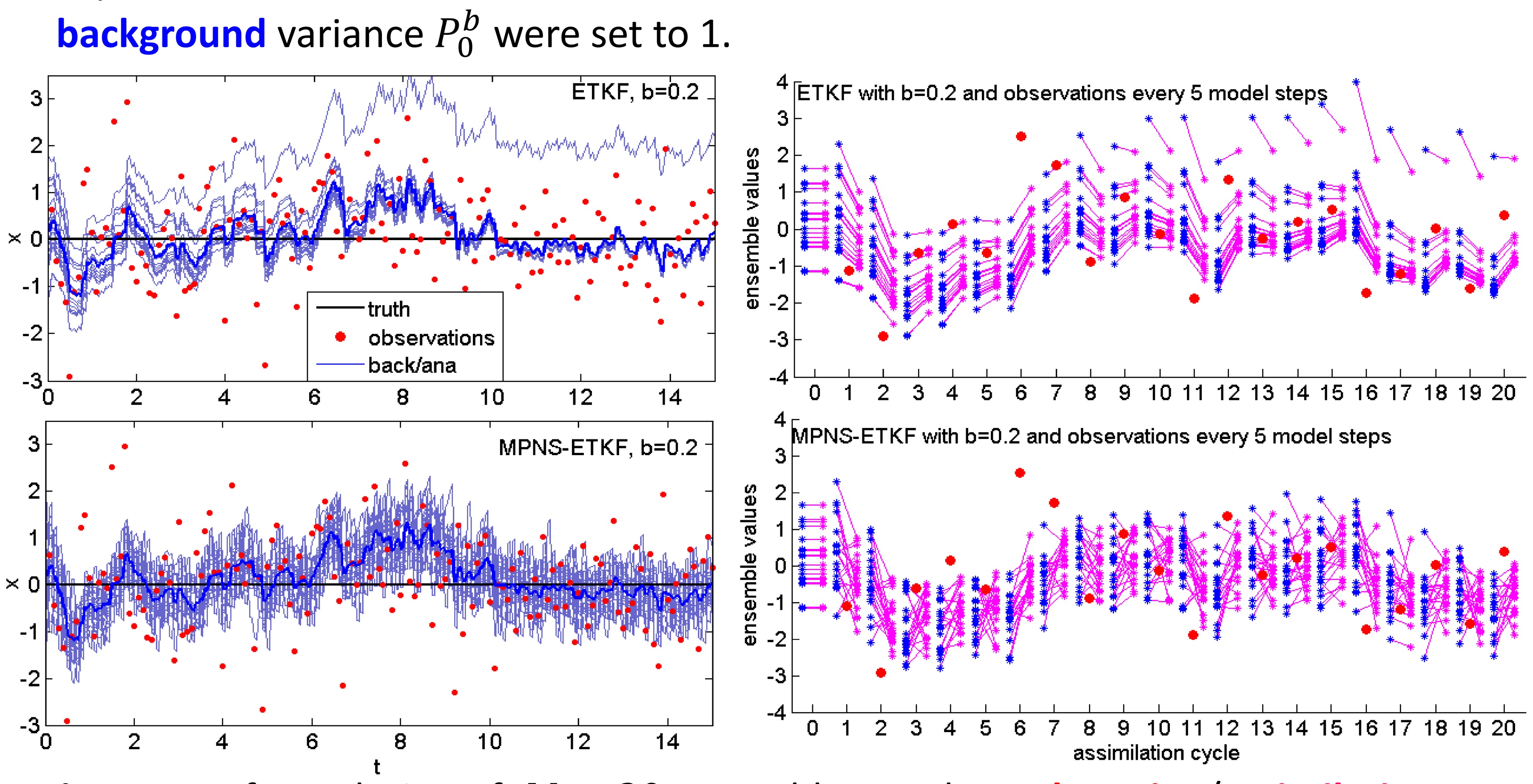

Figure 2. Left: evolution of $M = 20$ ensemble members **observing**/**assimilating** every 2Δ. The **ensemble clustering appears when using the symmetric ETKF** (top) **but not with the MPNS-ETKF** (bottom). Right: ensemble update at successive assimilation instants with **observation**/**assimilation** every 5Δ. For the symmetric **ETKF** (top), **the larger ensemble member progressively drifts away** (as a result of the nonlinear expansion in the forecast). For the **MPNS-ETKF** (bottom), the **constant mixing/resampling of ensemble members from background to analysis prevents the largest member from drifting away and the appearance of low order clustering.**

The **MPNS-ETKF** was satisfactorily **implemented in the Lorenz 1963 model.**

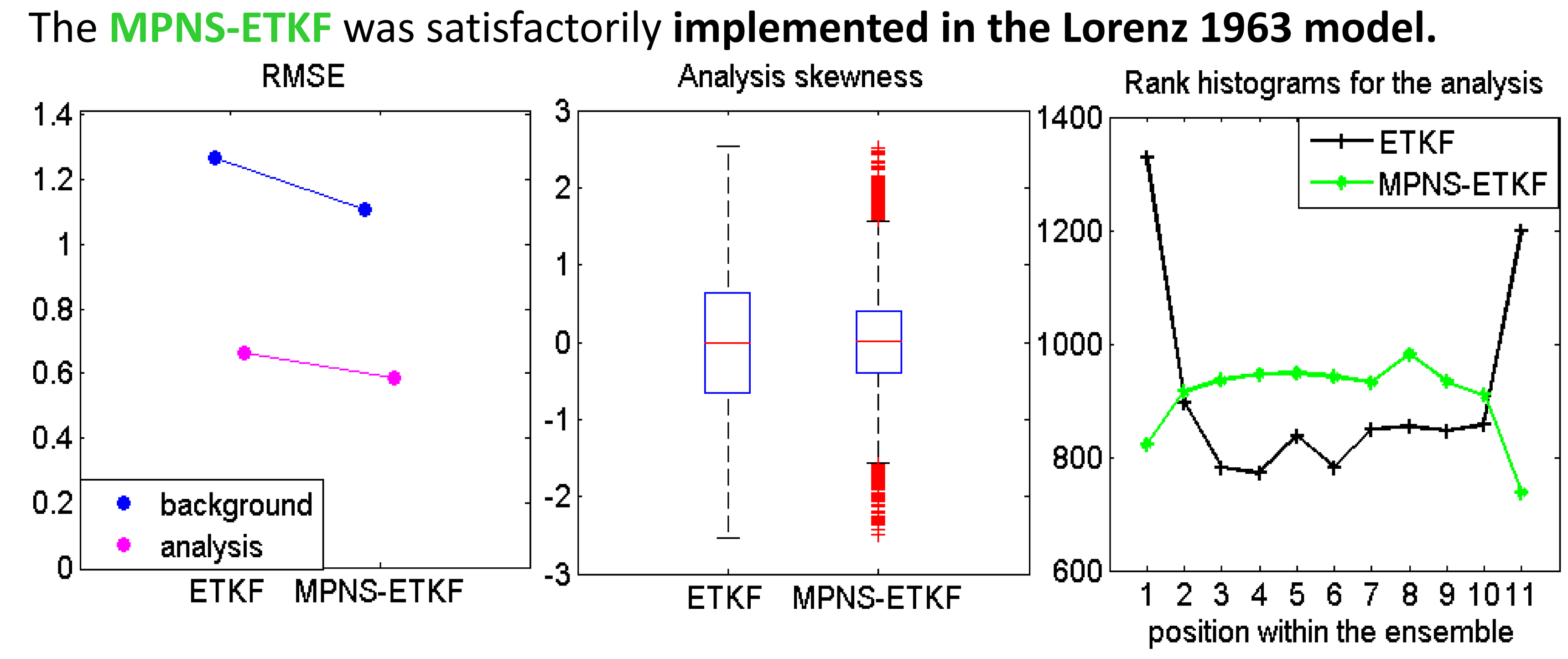

Figure 3. Experiments using the 3-variable Lorenz 1963 model with $M = 10$ ensemble members and $\mathbf{R} = 2\mathbf{I}$ over 10^4 analysis cycles . Left: the **mean RMSE** for both **background** and **analysis** are **reduced with the MPNS-ETKF**. Center: the **skewness values** of the **analysis** ensemble of the second variable; the **MPNS-ETKF yields more symmetric ensembles**. Right: **rank histograms** for the verification of the **analysis** ensembles with respect to the truth; **for the symmetric ETKF the truth often falls outside the ensemble**.

4. MNPS-ETKF with R-localization to avoid balance deterioration

In the application of the **ETKF** to a system with **large dimensions**, **R-localization must be imposed at individual grid points**. To preserve dynamic balance, **smooth transition of the ensemble weights from a grid point to next is essential**; **lack of smoothness results in loss of balance.** Among the ETKF family, the **LETKF** (Hunt *et al*, 2007) **is the only method that guarantees this smoothness** by the use of symmetric $\mathbf{W}^a_{LETKF}$ (see Section 2).

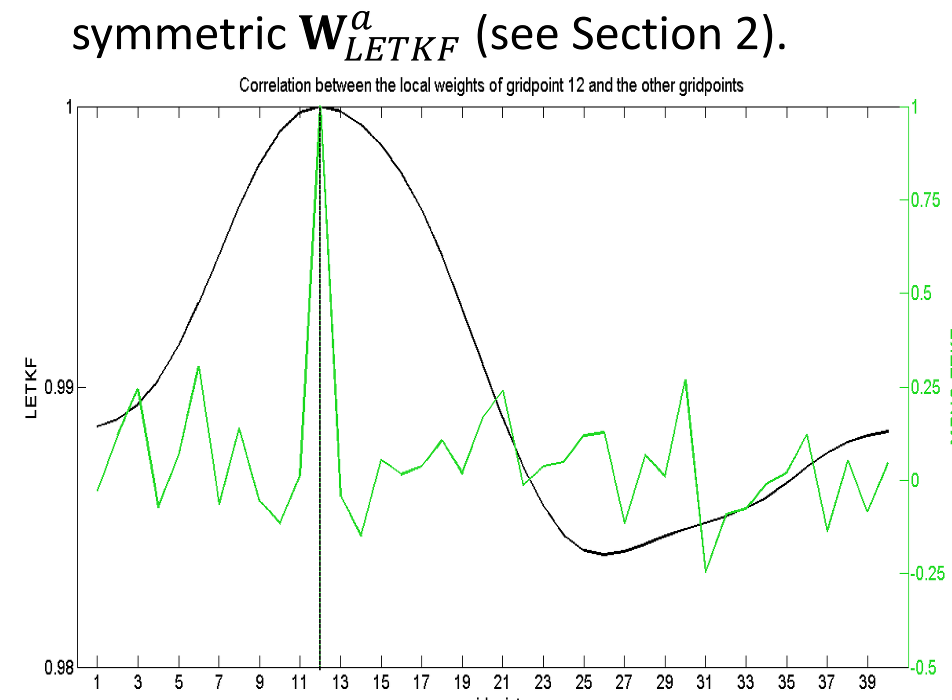

Figure 4. Experiment using the 40 variable Lorenz 1996 model with $M = 20$ ensemble members and **R-localization** with a localization radius $\lambda = 4$. The **correlation of the local analysis weights at gridpoint with the weights in the rest of the gridpoints** is shown for the **LETKF analysis** and **MPNS-ETKF analysis**. Only the **symmetric ETKF guarantees a smooth transition.**

To **avoid the ensemble clustering** (Section 3), a **global rotation $\mathbf{U}$** can be **applied to the LETKF global analysis** for improved performance:

$$[\mathbf{X}^a_{MPNS-ETKF}]_{global} = [\mathbf{X}^a_{LETKF}]_{global}\mathbf{U}^{\mathrm{T}}$$

Where $\mathbf{U} \in \Re^{M\times M}$ must be orthonormal $\mathbf{U}^{\mathrm{T}}\mathbf{U} = \mathbf{I}$ and $\mathbf{U}^{\mathrm{T}}\mathbf{1} = \mathbf{1}$ (e.g. Sakov and Oke, 2008). This **locally-symmetric globally-non-symmetric analysis** scheme was **implemented in the Lorenz 1996 model with satisfactory results**.

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