On ensemble forecasts, singular vectors and reliability

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ECMWF

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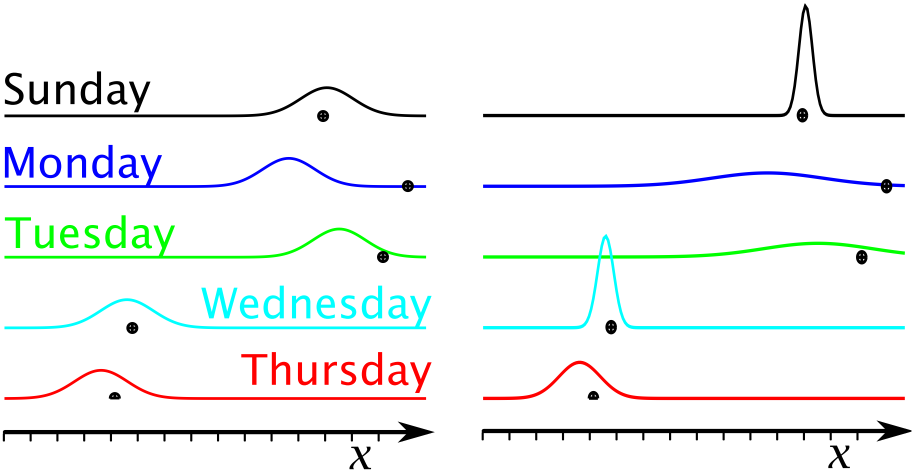
Acknowledgements: Franco Molteni, Simon Lang

Monday blues



- resolution/sharpness
- reliability

Reliability of variances

- reliability requires that the ensemble variance matches the ensemble mean error variance (sufficient sample size)
- this statistical consistency has to hold (in principle) everywhere
- can look at different locations, spatial scales, variables
- **here:** quantify variances in subspaces spanned by sets of (evolved) singular vectors (SVs)

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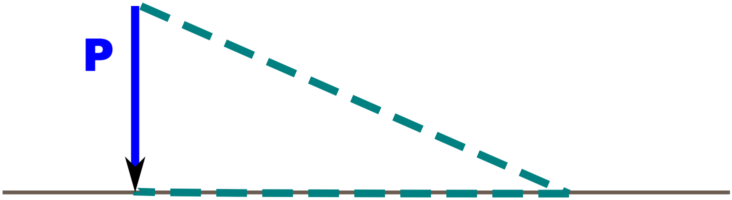
why?

- Enough variance in directions that are dynamically the most sensitive?
- SV as initial perturbations: diagnose consistency of initial uncertainty representation
- Technique is applicable to any ensemble regardless of perturbation technique

Methodology

- for verification of forecast valid at t_1 and initialized at t_0 compute SVs that grow from t_0 to t_1.
- define operator **P** that projects on the subspace spanned by the evolved SVs (valid at t_1).



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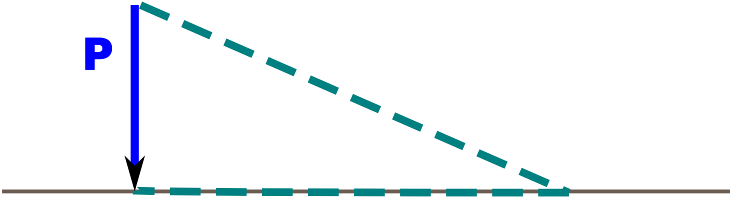


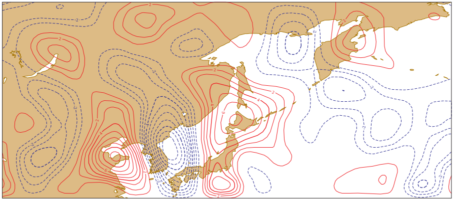
- project error of ensemble mean into space spanned by SVs and compute error variance
- project perturbations about ensemble mean into space spanned by SVs and compute ensemble variance

Projection example

48-hour ens. mean error: 200–500 hPa meridional wind

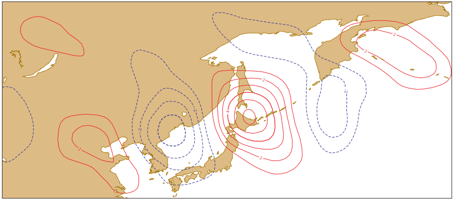
full error

error ensemble mean



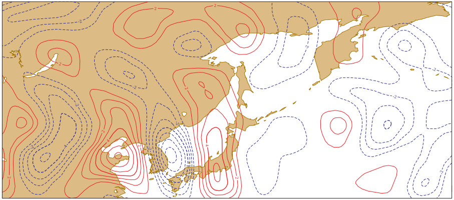
subspace of leading SVs

error ensemble mean



orthogonal complement

error ensemble mean

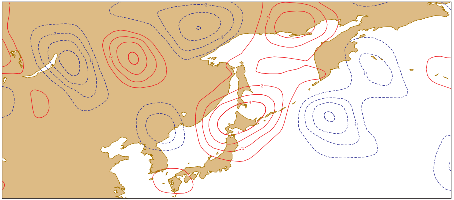


Projection example

48-hour perturbation member 2: 200–500 hPa meridional wind

full perturbation

perturbation member 2



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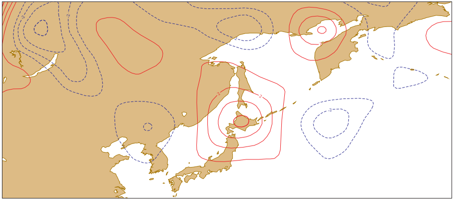

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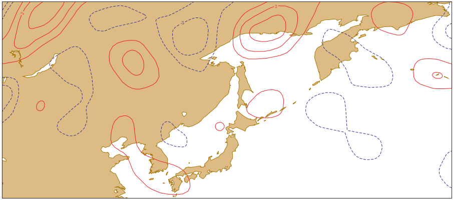

Methodology (II)

The initial SVs $\mathbf{v}_i$ are solutions of

$$\mathbf{M}^{\mathrm{T}}\mathbf{L}^{\mathrm{T}}\mathbf{E}\mathbf{L}\mathbf{M}\mathbf{v}_i = \sigma_i^2 \mathbf{E}\mathbf{v}_i \qquad (1)$$

- **M** propagator from t_0 to t_1
- **L** local projection operator (e.g. Northern Extra-tropics sfc–100 hPa)
- **E** symm. pos. def. matrix; initial and final metric
- $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \ldots$ are the singular values

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The diagnostics requires the normalized, evolved and projected SVs:

$$\mathbf{w}_i = \sigma_i^{-1}\mathbf{L}\mathbf{M}\mathbf{v}_i \tag{2}$$

Methodology (III)

The projection operator

Now, consider projection on space spanned by vectors $\{\mathbf{w}_i | i \in I\}$

$$\mathbf{P}_I = \sum_{i \in I} \mathbf{w}_i \mathbf{w}_i^{\mathrm{T}} \mathbf{E} \quad \text{where} \quad I = \{i_1, i_2, \ldots, i_M\} \tag{3}$$

For any vector x, the projection is

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space	notation
$I = \{1, 2, \ldots, 50\}$	SV1–50
$I = \{51, 52, \ldots, N\}$	C(SV1–50)
$I = \{21, 22, \ldots, 30\}$	SV21–30

N is the dimension of the SV state space.

Numerical experiments with the ECMWF EPS

- Integrated Forecasting System (IFS)
- SV-diagnostic using leading 50 extra-tropical SVs (same configuration as in operational EPS):
  - T42 resolution, 62 level, dry TL model, $N = 3.5 \times 10^5$
  - total energy norm at initial and final time
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- ensemble forecasts
  - resolution 32 km (T_L639)
  - 20 member
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    - ⋆ Reduced stdev $0.48A$ (from Nov. 2010)

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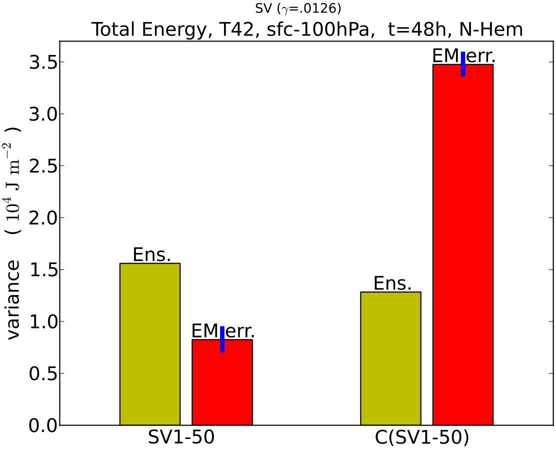
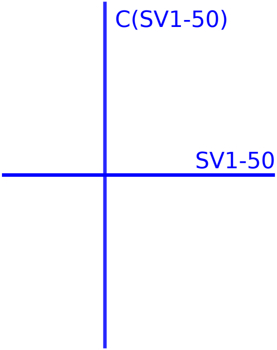
  - operational EPS configuration since November 2010 (36R4)
    - initial SVs ($0.48A$)
    - perturbations from an ensemble of 4D-Vars with perturbed obs (EDA)
    - representations of model uncertainty (SPPT and SKEB)
- 26 start dates: 14 August – 3 October 2008, 0 UTC, every 48 h

Only SV initial perturbations

amplitude A used operationally in 36R2





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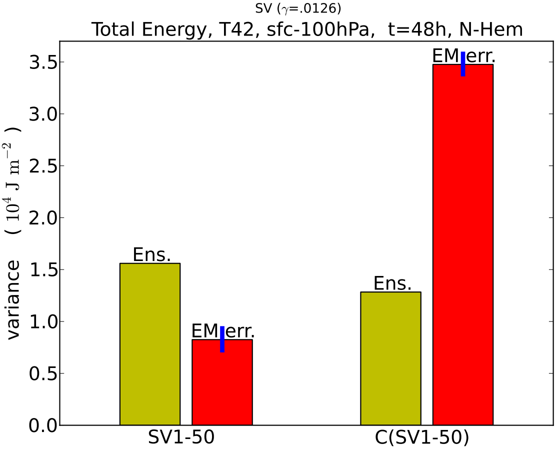

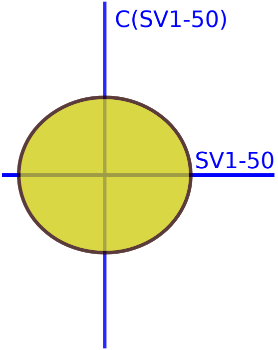

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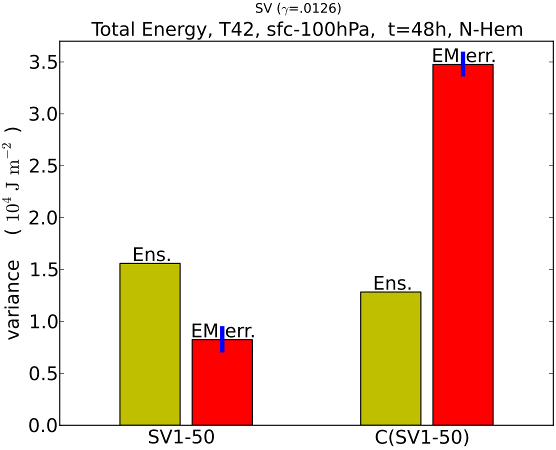
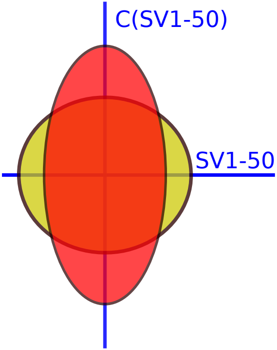
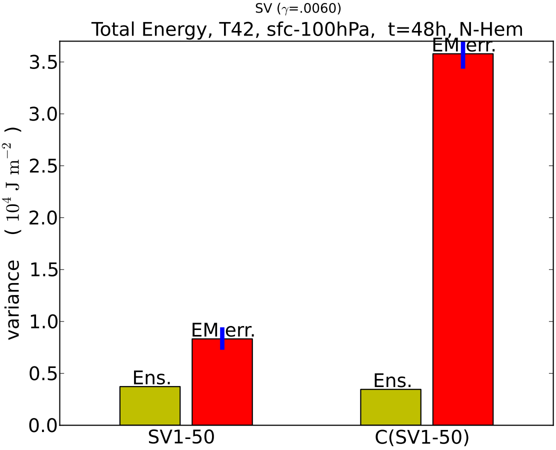
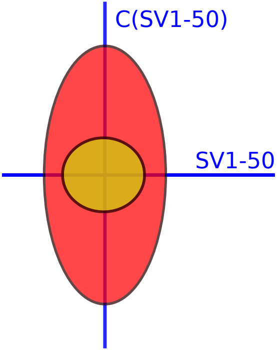




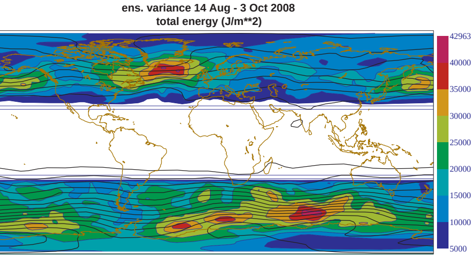
Only SV initial perturbations

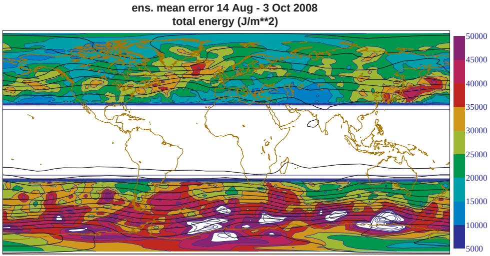
amplitude 0.48A used operationally in 36R4



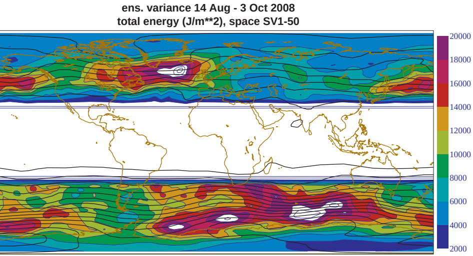


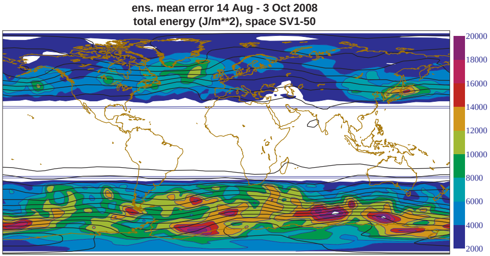
Geographical distribution: Full space



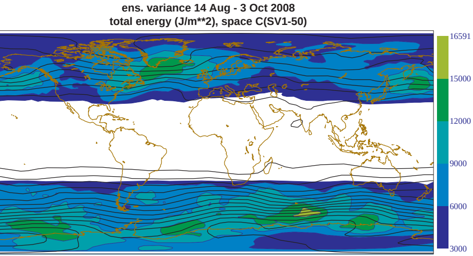


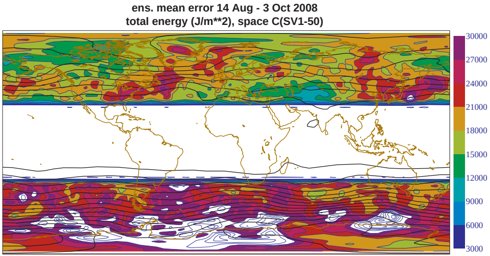
Geographical distribution: SV1–50





Geographical distribution: C(SV1–50)



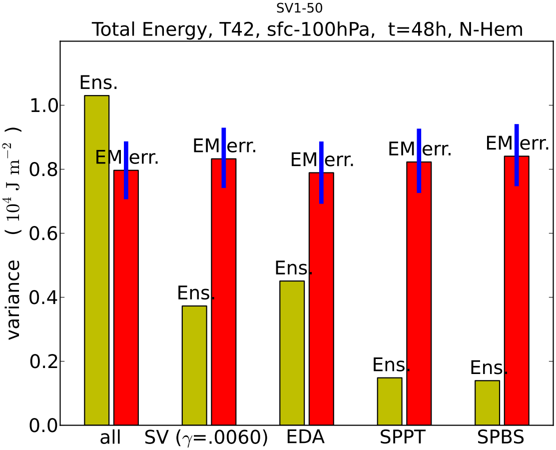


Numerical experiments with the ECMWF EPS (II)

- 2 experiments with SV perturbations *only*:
  - Large stdev A (as used before Nov. 2010)
  - Reduced stdev $0.48A$ (from Nov. 2010)
- operational EPS configuration since November 2010 (36R4)
  - initial SVs ($0.48A$)
  - perturbations from an ensemble of 4D-Vars with perturbed obs (EDA)
  - representations of model uncertainty (SPPT and SKEB)
- four experiments: SVs only ($0.48A$), EDA only, SPPT only, SKEB only
- 26 start dates: 14 August – 3 October 2008, 0 UTC, every 48 h

Operational EPS configuration (since Nov. 2010)

Contributions from the four sources of uncertainties: SV1–50



Operational EPS configuration (since Nov. 2010)

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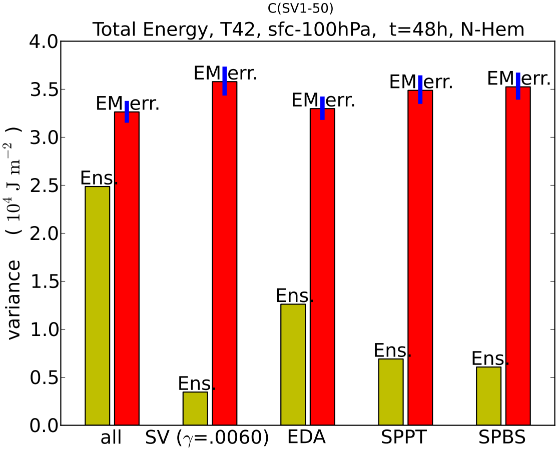
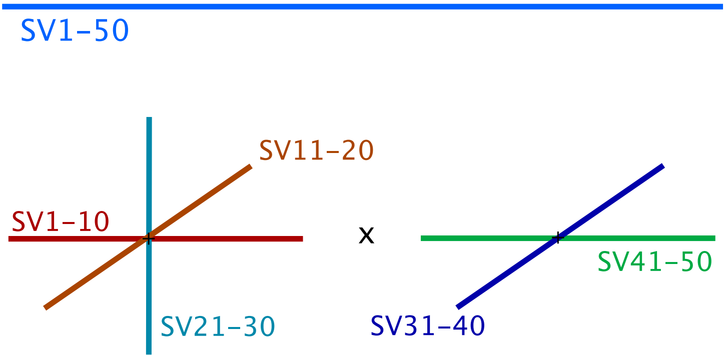


Lower-dimensional SV subspaces

stratify by singular vector growth



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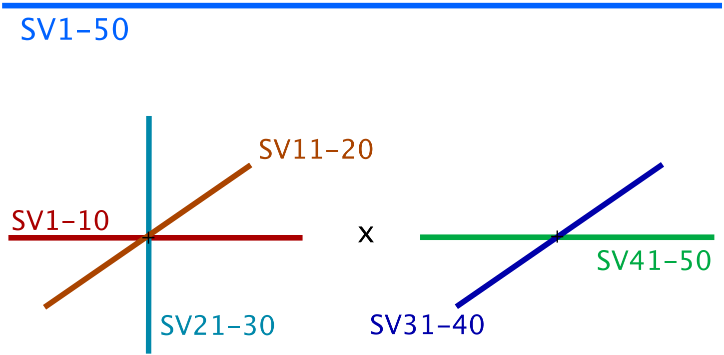
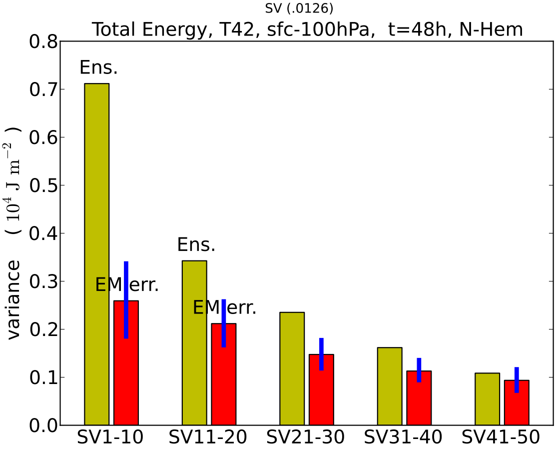


return to experiment with SVs only (amplitude A)

Spread and error stratified by SV growth

SV perturbations only — amplitude A — Northern Extra-tropics



Spread and error stratified by SV growth

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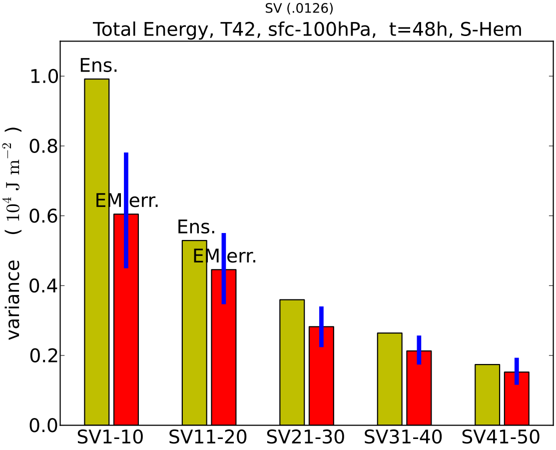

Discussion

How to further improve/extend diagnostic?

- separate error into bias and random component
- account for analysis uncertainties
- sensitivity to SV configuration
- tropical cyclones
- tropics
- include probabilistic scores
- seasonal variations

Discussion (II)

How to exploit diagnostic to improve ensemble forecast systems?

- Explained variance by first 50 extra-tropical SVs rather small:
  - try many more SVs?
- Overdispersion larger for larger SV-growth. Is it possible to improve the scaling in the Gaussian sampling?
  - empirical (based on past verification) *or*
  - using (possibly calibrated) flow-dependent analysis error variances from EDA
- Explore alternative SV configurations for the initial perturbations
  - resolution, optimisation time, physics in TL/AD
  - initial metric
- Explore alternative EDA configurations
  - representation of model uncertainties
  - calibration
  - ...

Summary

- Diagnosing spread and error in SV subspaces can help to quantify deficiencies arising from the design of the initial perturbations.
- Better understanding of the limitations of using (only) SV-based initial perturbations
- The revision of the EPS perturbation methodology in November 2011 (halving of the SV perturbation amplitude, 3-scale SPPT and SKEB) improved significantly the spread error relationship in the subspace spanned by the leading 1–50 extra-tropical SVs and its orthogonal complement.