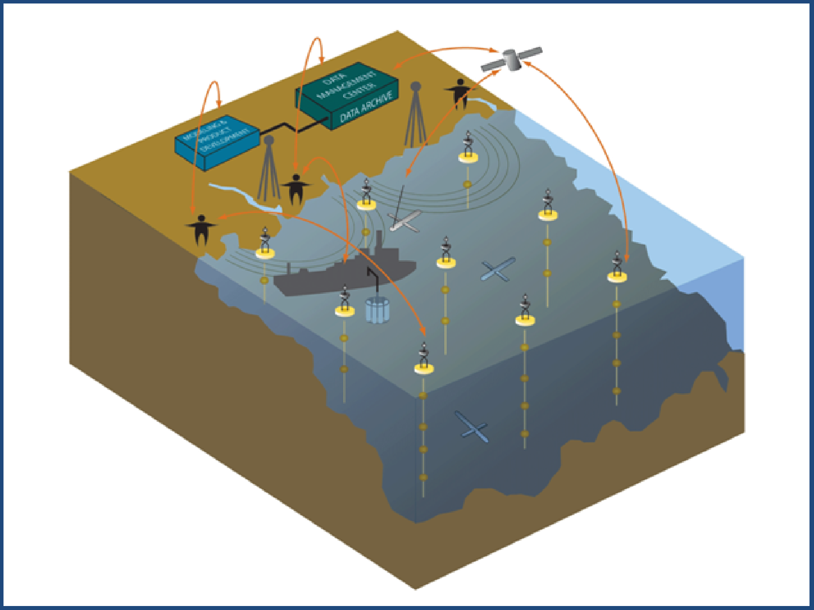
A Multi-Scale Three-Dimensional Variational Data Assimilation Scheme and Its Application to Coastal Oceans



Zhijin Li

Jet Propulsion Laboratory, California Institute of Technology

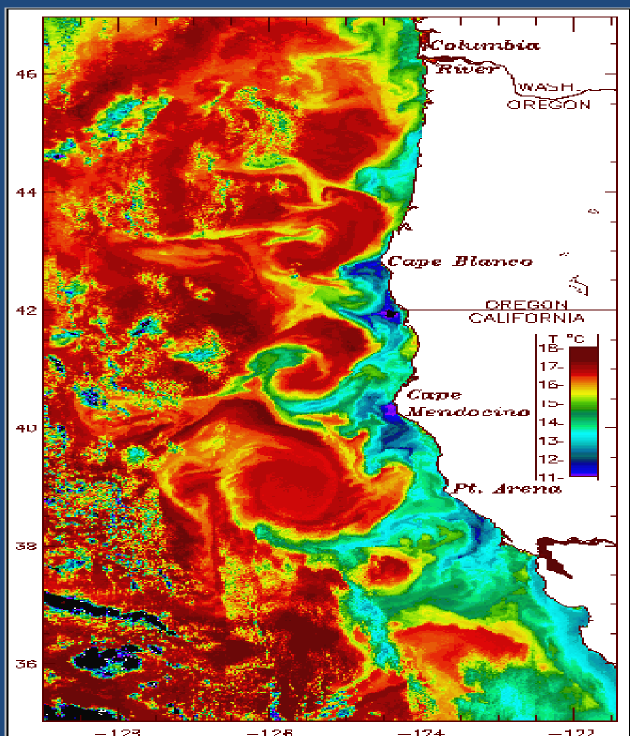
The 9th Workshop on Adjoint Model Applications in Dynamic Meteorology

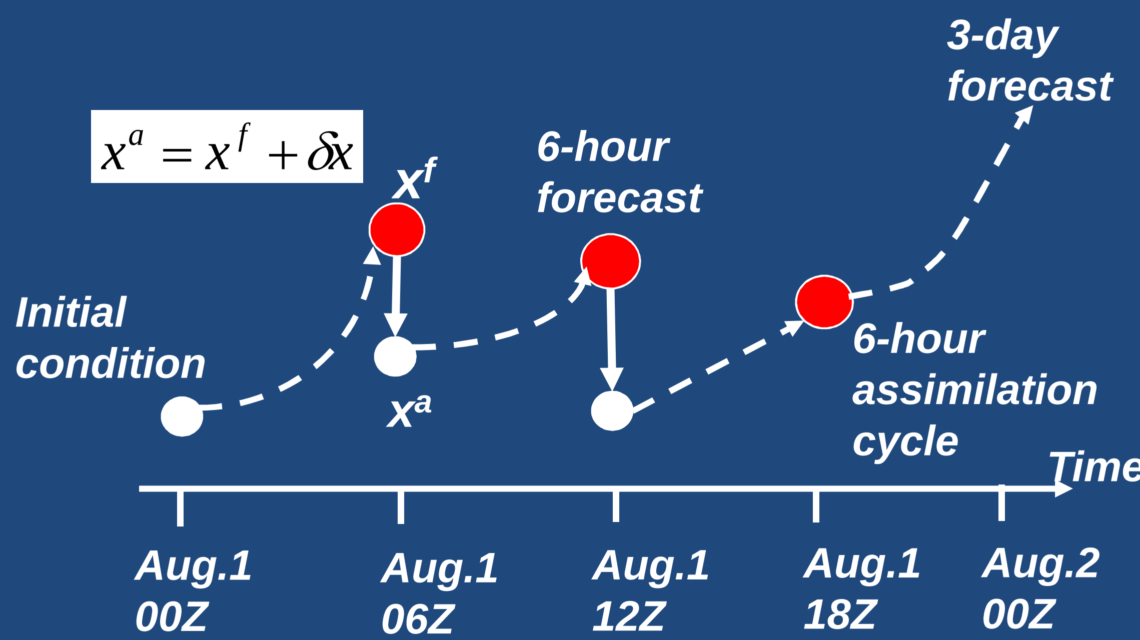
Cefalu, Sicily, Italy, 10-14 October 2011

Acknowledgements

- Dr Yi Chao and his group (JPL)
- Prof James C McWilliams and his group (UCLA)
- Prof Kayo Ide (UMD)
- We acknowledge the support from NASA Physical Oceanography Program

3DVAR Data Assimilation and Forecast Cycle

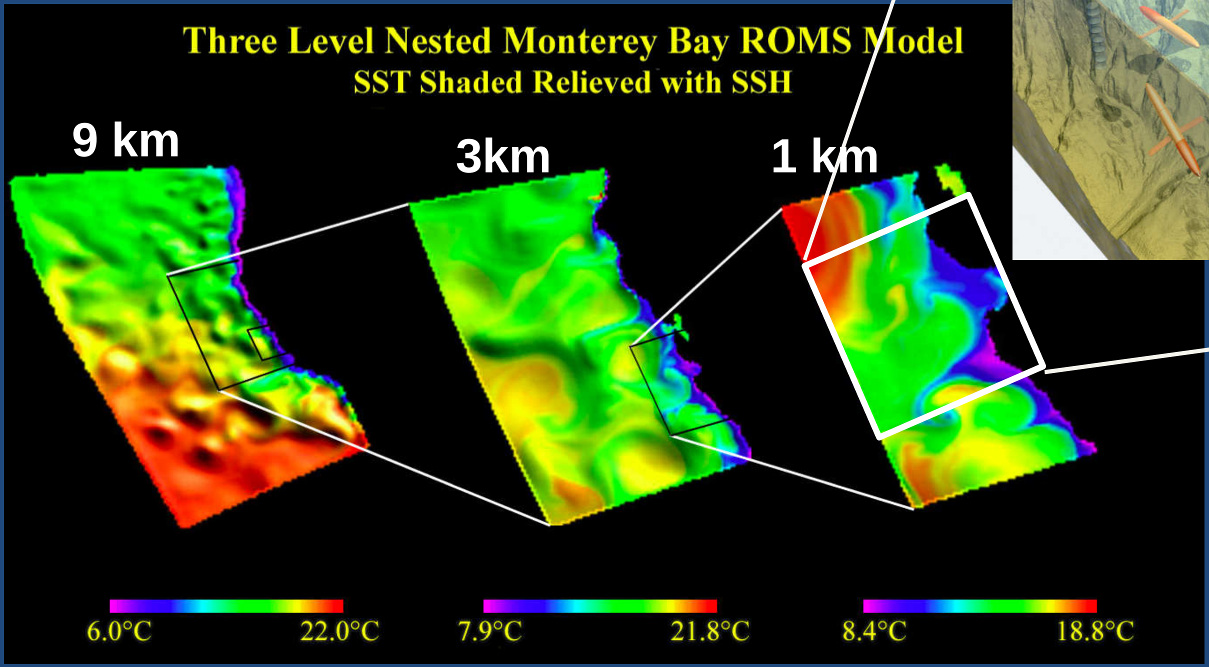


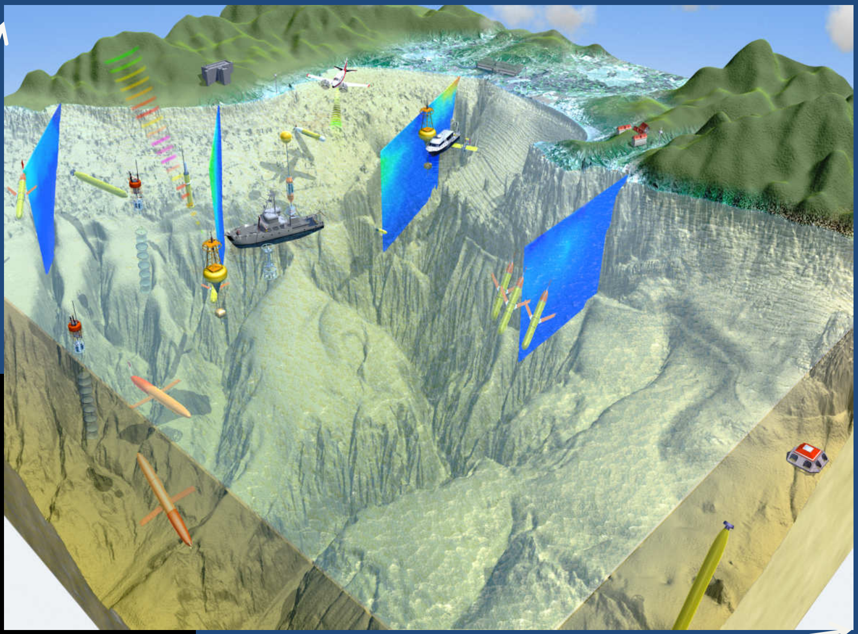


- Diurnal variation
- Rapid response to wind stresses
- Eddies, fronts, filaments, etc

Autonomous Ocean Sampling Network (AOSN) Experiment August, 2003

"Bring together sophisticated new robotic vehicles with advanced ocean models to improve our ability to observe and predict the ocean"





www.mbari.org/aosn

An Incremental There-Dimensional Variational Data Assimilation (3DVAR)

$$\min_x J(x) = \frac{1}{2}(x - x^f)^T B^{-1}(x - x^f) + \frac{1}{2}(Hx - y)^T R^{-1}(Hx - y)$$

$$\min_x J(\delta x) = \frac{1}{2}\delta x^T B^{-1}\delta x + \frac{1}{2}(H\delta x - \delta y)^T R^{-1}(H\delta x - \delta y)$$

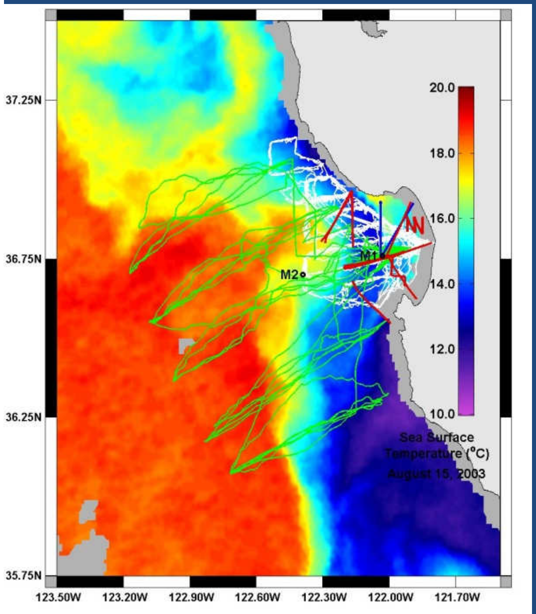
$$\delta y = y - Hx^f$$

1. **Real-time capability**
2. **Implementation with sophisticated and high resolution model configurations**
3. **Flexibility to assimilate various observation simultaneously**

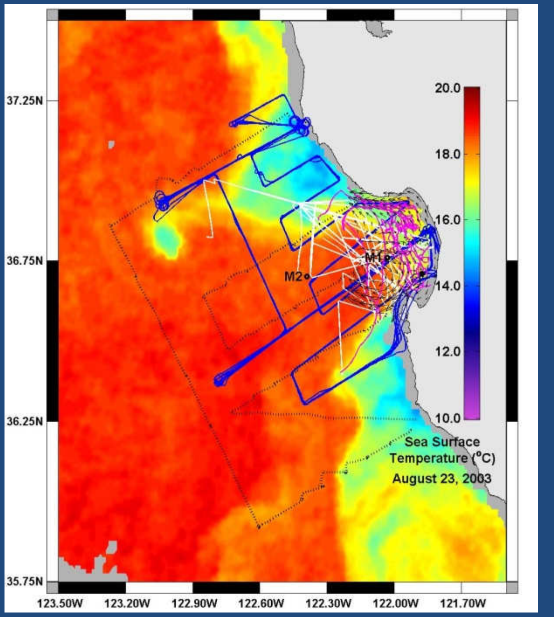
(Li et al., 2006, MWR; Li et al., 2008, JGR)

AOSN Intensive Observations

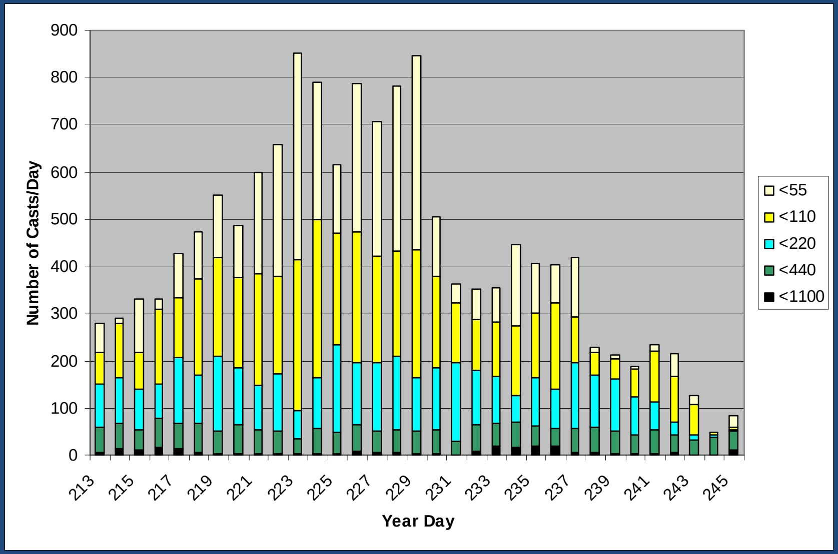
- T/S profiles from gliders
- Ship CTD profiles
- Aircraft SSTs
- AUV sections
- HF radar velocities



Glider and AUV tracks



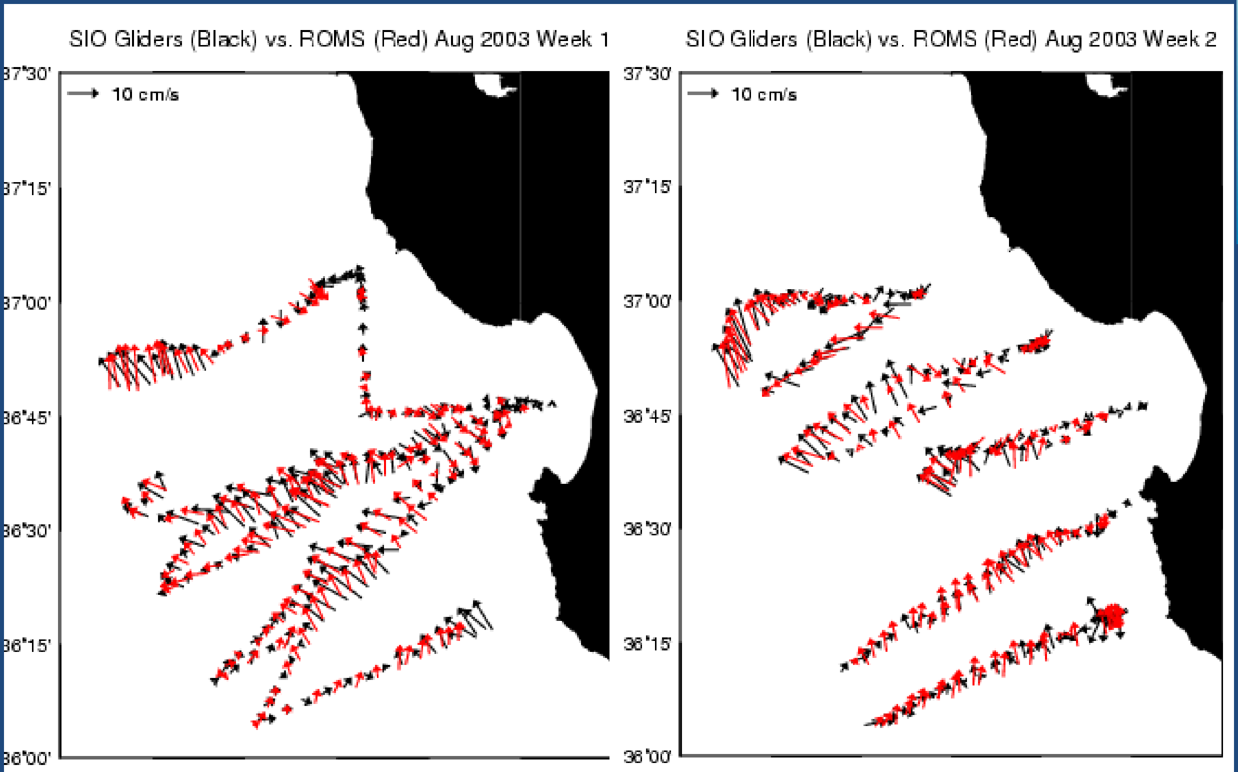
Ships, Aircrafts, and HF radars

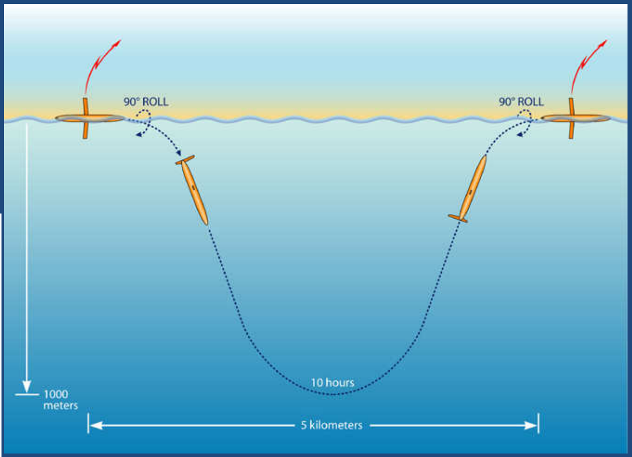


T/S Profile Data

Performance of ROMS3DVAR August 2003

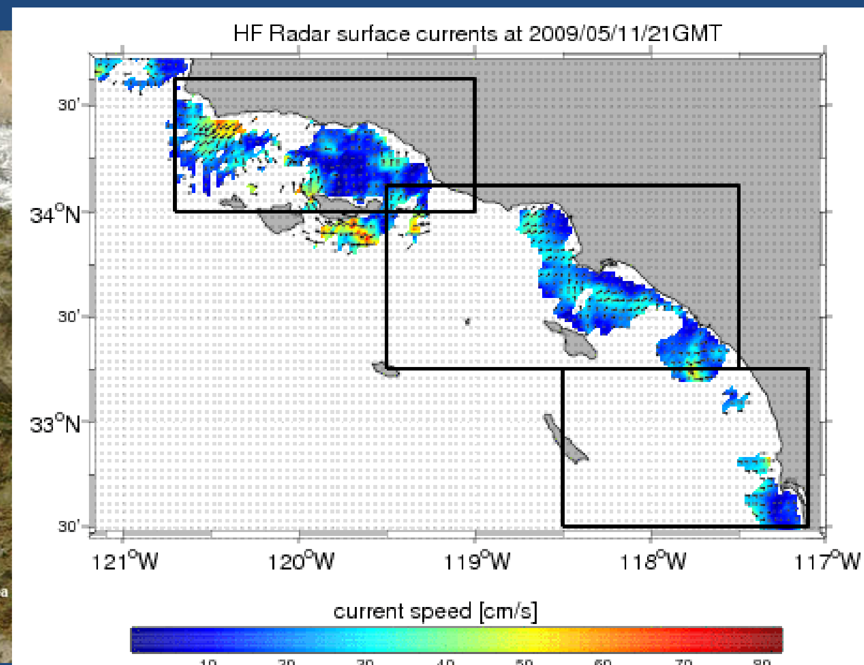
Comparison of Glider-Derived Currents (vertically integrated current)





(Chao and Li et al., 2009, DSR)

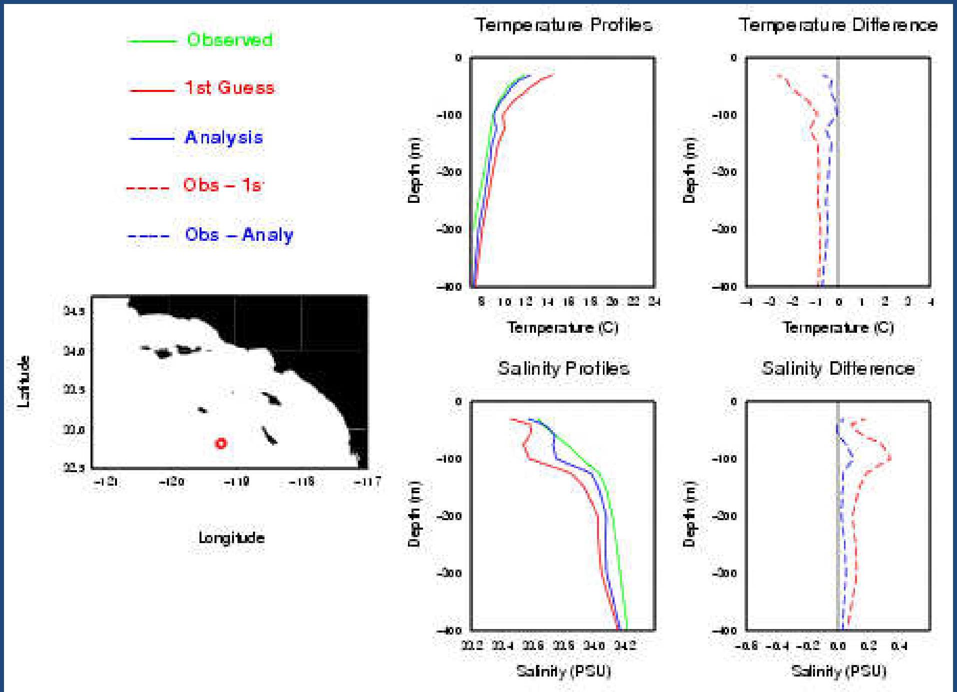
Southern California Coastal Ocean Observing System (SCCOOS)

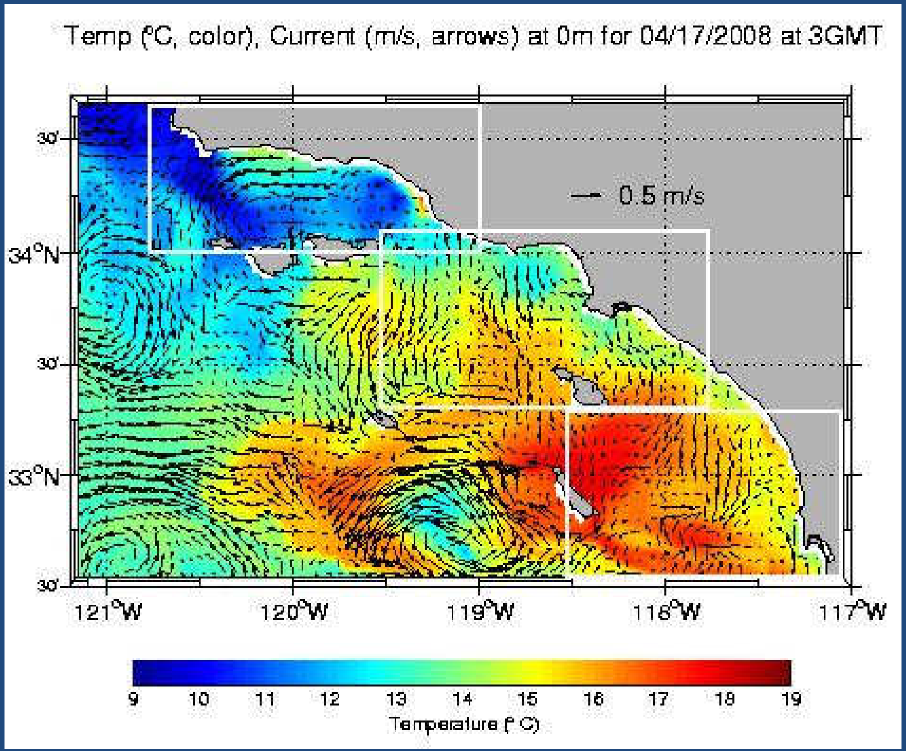


Decorrelation length scales: 15-50km

Challenge: Assimilating sparse vertical profiles along with high resolution observations for a very high resolution model

Challenges with 3DVAR





Fourier Series Expansion of Homogenous Errors

$$e(x) = \sum_{n=-\infty}^{\infty} e_n \exp(inx)$$

$$e_n = \frac{1}{2\pi}\int_{-\infty}^{\infty} e(x)\exp(-inx)\,dx$$

$$\langle e_m e_n^* \rangle = \begin{cases} = 0, m \neq n \\ = c_n, m = n \end{cases}$$

Wiener-Khintchine Theorem

$$c(r) = \langle e(x)e(x+r) \rangle$$

Error Covariance

$$c_n = \frac{1}{2\pi}\int_{-\infty}^{\infty} c(r)\exp(-inr)dr$$

$$c(r) = \int_{-\infty}^{\infty} c_n \exp(inr)dn$$

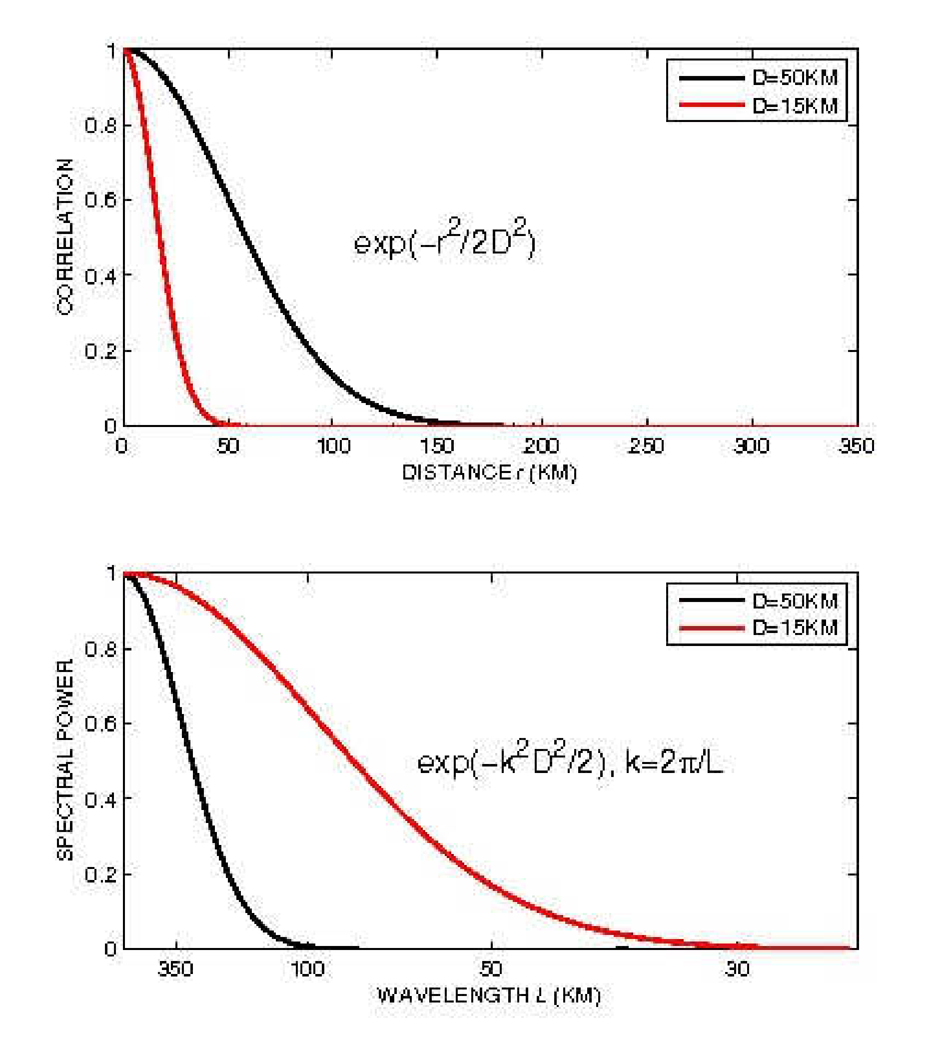
$$c_n = e_n^2$$

power spectral density

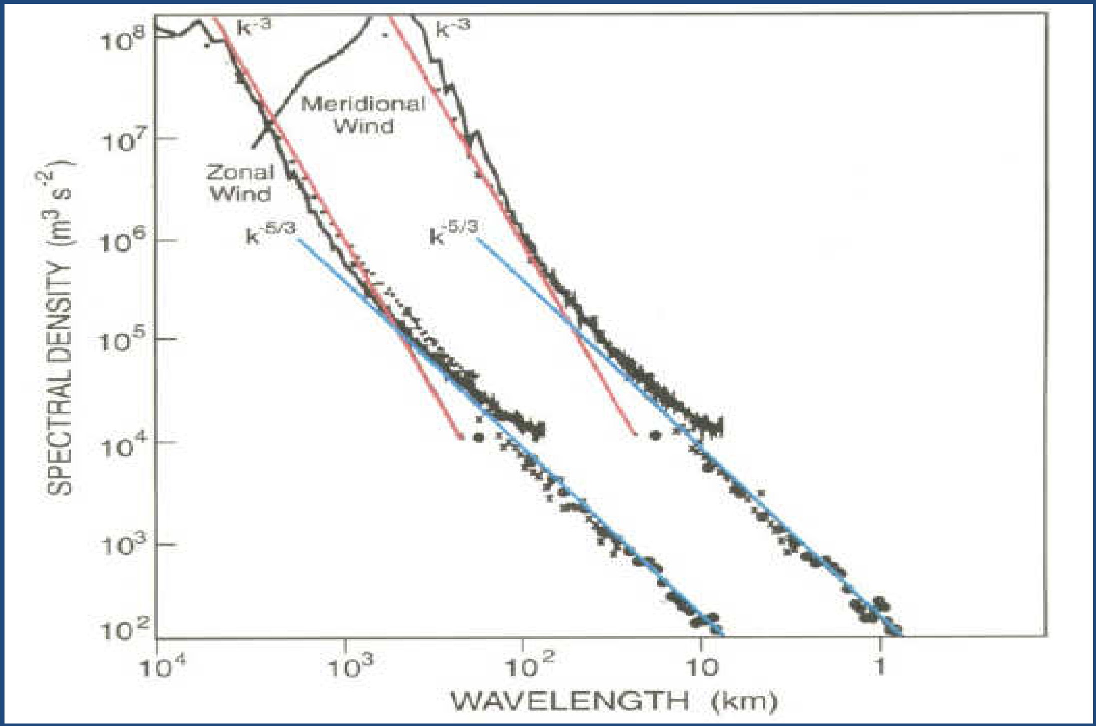
Error Covariance in 3DVAR: Smoothing and Spreading

$$c(r) = e^{-\frac{r^2}{2D^2}}$$

$$c_n = \frac{D}{\sqrt{2\pi}} \exp\left(-\frac{n^2 D^2}{2}\right)$$



A Multi-Decorrelation Length Scale Scheme for High Resolution Models?



Background Error

$$x = x_L + x_S$$

$$e = e_L + e_S$$

$$\langle e_L e_S^T \rangle = 0$$

$$B = B_L + B_S$$

3DVAR with a Background Error Covariance of Multi-Decorrelation Length Scales

$$\min_{x} J(\delta x) = \frac{1}{2}\delta x^T \left(B_L + B_S\right)^{-1}\delta x + \frac{1}{2}(H\delta x - \delta y)^T R^{-1}(H\delta x - \delta y)$$

$$\min_{\delta x_L} J(\delta x_L) = \frac{1}{2}\delta x_L{}^T B_L^{-1}\delta x_L + \frac{1}{2}(H\delta x_L - \delta y)^T \left(HB_S H^T + R\right)^{-1}(H\delta x_L - \delta y)$$
$$\min_{\delta x_S} J(\delta x_S) = \frac{1}{2}\delta x_S{}^T B_S^{-1}\delta x_S + \frac{1}{2}(H\delta x_S - \delta y)^T \left(HB_L H^T + R\right)^{-1}(H\delta x_S - \delta y)$$

$$p(x_L \mid y)$$
$$p(x_S \mid y)$$

(Lorenc, 1986)

(Li et al., 2011, QJRMS, in revision)

Multi-Scale Representativeness Errors

Observational Error Covariance for Large Scale

$$R + HB_S H^T$$

$$
\begin{aligned}
e_L^o &= \delta y - H\delta x_L^t \\
&= \left(y - y^t\right) - \left(Hx^t - y^t\right) - H\left(x_S^b - x_S^t\right) \\
&= e^{om} + e^{or} + e_S^{or}
\end{aligned}
$$

Measurement error + Representativeness error + Multi-scale representativeness error

Multi-Scale Data Assimilation

High resolution Observation

$$y^h = y_L^h + y_S^h$$

Multi-scale DA

$$\min_{\delta x_L} J(\delta x_L) = \frac{1}{2}\delta x_L{}^T B_L^{-1}\delta x_L + \frac{1}{2}(H\delta x_L - \delta y_L^h)^T R_L^{-1}(H\delta x_L - \delta y_L^h)$$

$$\min_{\delta x_S} J(\delta x_S) = \frac{1}{2}\delta x_S{}^T B_S^{-1}\delta x_S + \frac{1}{2}(H\delta x_S - \delta y_S^h)^T R_S^{-1}(H\delta x_S - \delta y_S^h)$$

3DVAR Formulations

3DVAR

$$\min_{x} J = \frac{1}{2}(x - x^{f})^{T} B^{-1}(x - x^{f}) + \frac{1}{2}(Hx - y)^{T} R^{-1}(Hx - y)$$

AB-3DVAR

$$\min_{\delta x_L} J = \frac{1}{2}\delta x_L{}^{T} B_L^{-1}\delta x_L + \frac{1}{2}(H\delta x_L - \delta y)^{T}\left(HB_S H^{T} + R\right)^{-1}(H\delta x_L - \delta y)$$

$$\min_{\delta x_S} J = \frac{1}{2}\delta x_S{}^{T} B_S^{-1}\delta x_S + \frac{1}{2}(H\delta x_S - \delta y)^{T}\left(HB_L H^{T} + R\right)^{-1}(H\delta x_S - \delta y)$$

MS-3DVAR

$$\min_{\delta x_L} J = \frac{1}{2}\delta x_L{}^{T} B_L^{-1}\delta x_L + \frac{1}{2}(H\delta x_L - \delta y_L^{h})^{T} R_L^{-1}(H\delta x_L - \delta y_L^{h})$$

$$\min_{\delta x_S} J = \frac{1}{2}\delta x_S{}^{T} B_S^{-1}\delta x_S + \frac{1}{2}(H\delta x_S - \delta y_S^{h})^{T} R_S^{-1}(H\delta x_S - \delta y_S^{h})$$

Experiments with Idealized Problems

True State

$$x_n^t = S_0 \sum_{k=1}^{K} a_k^t \cos\left(\frac{k\pi n}{N} + \phi_k^t \right)$$

$$a_k^t = k^{\gamma}$$

$$\phi_k^t = \alpha_k \pi, \alpha_k \in (-1,1)$$

$$N = 200, K = N/5$$

Background/First Guess

$$x_n^b = S_0 \sum_{k=1}^{K} a_k^b \cos\left(\frac{k\pi n}{N} + \phi_k^t \right)$$

$$a_k^b = \beta^k a_k^t$$

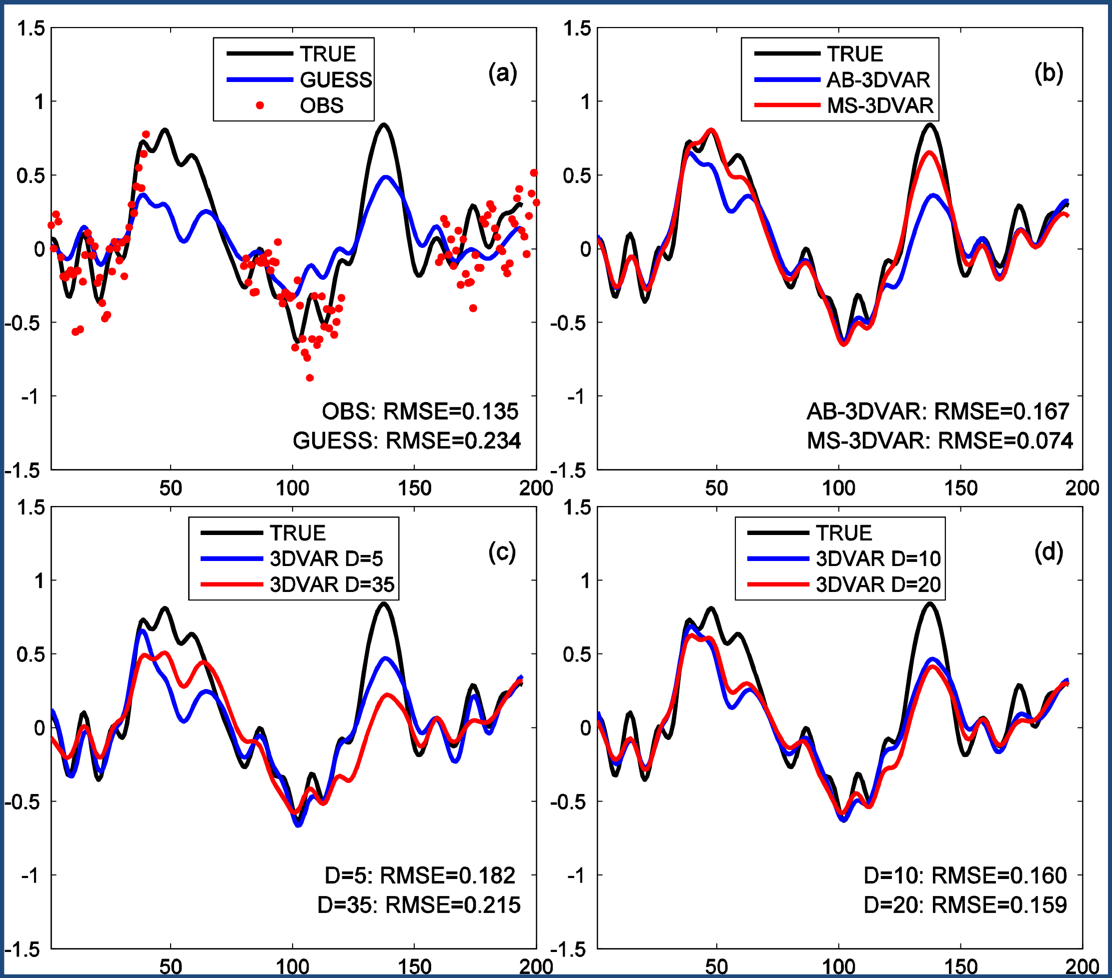
Observations

$$y^o = x_n^t + a_e^o e^o$$

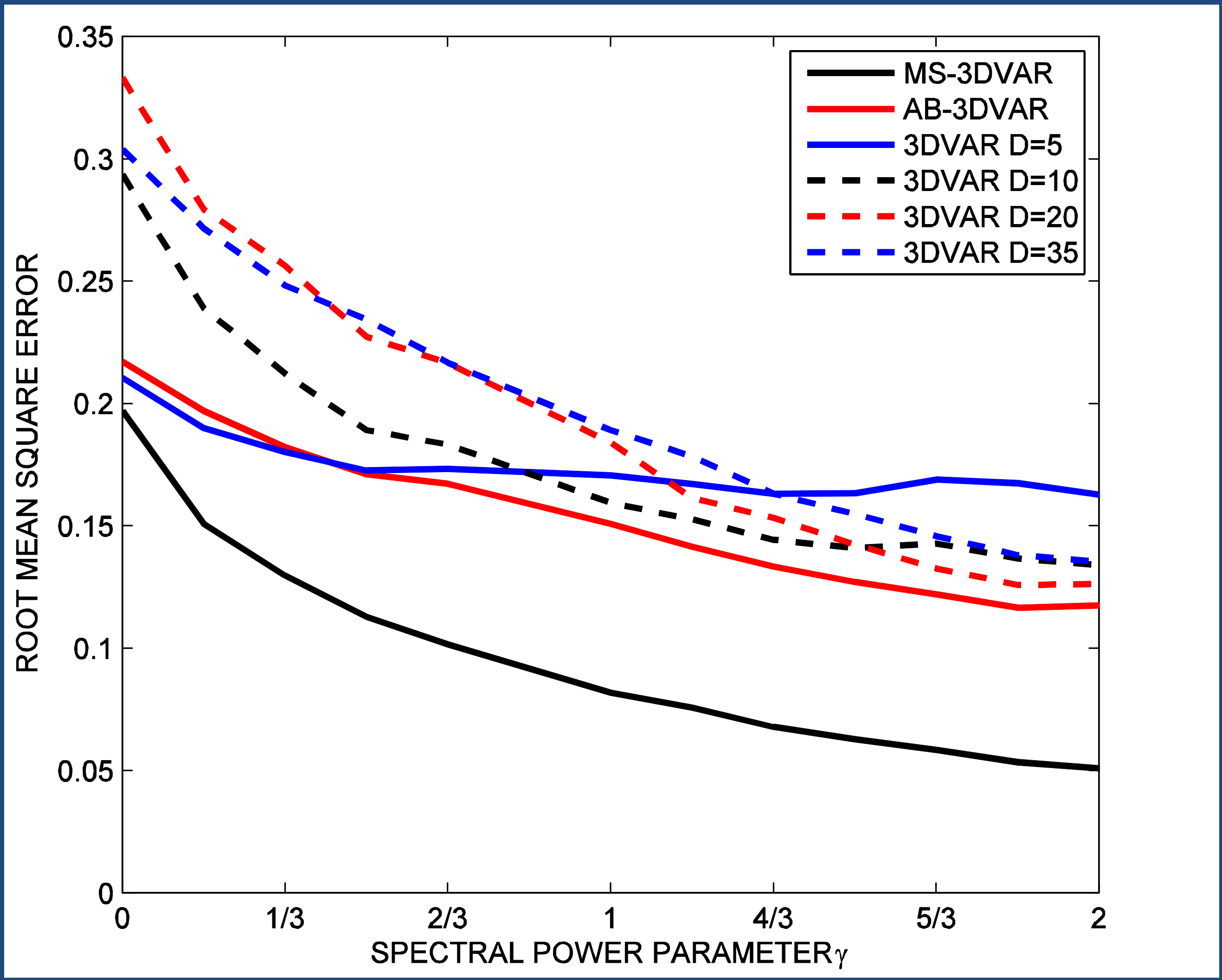
Difference between SD-3DVAR, MD-3DVAR, and MS-3DVAR Solutions

Patchy Observation

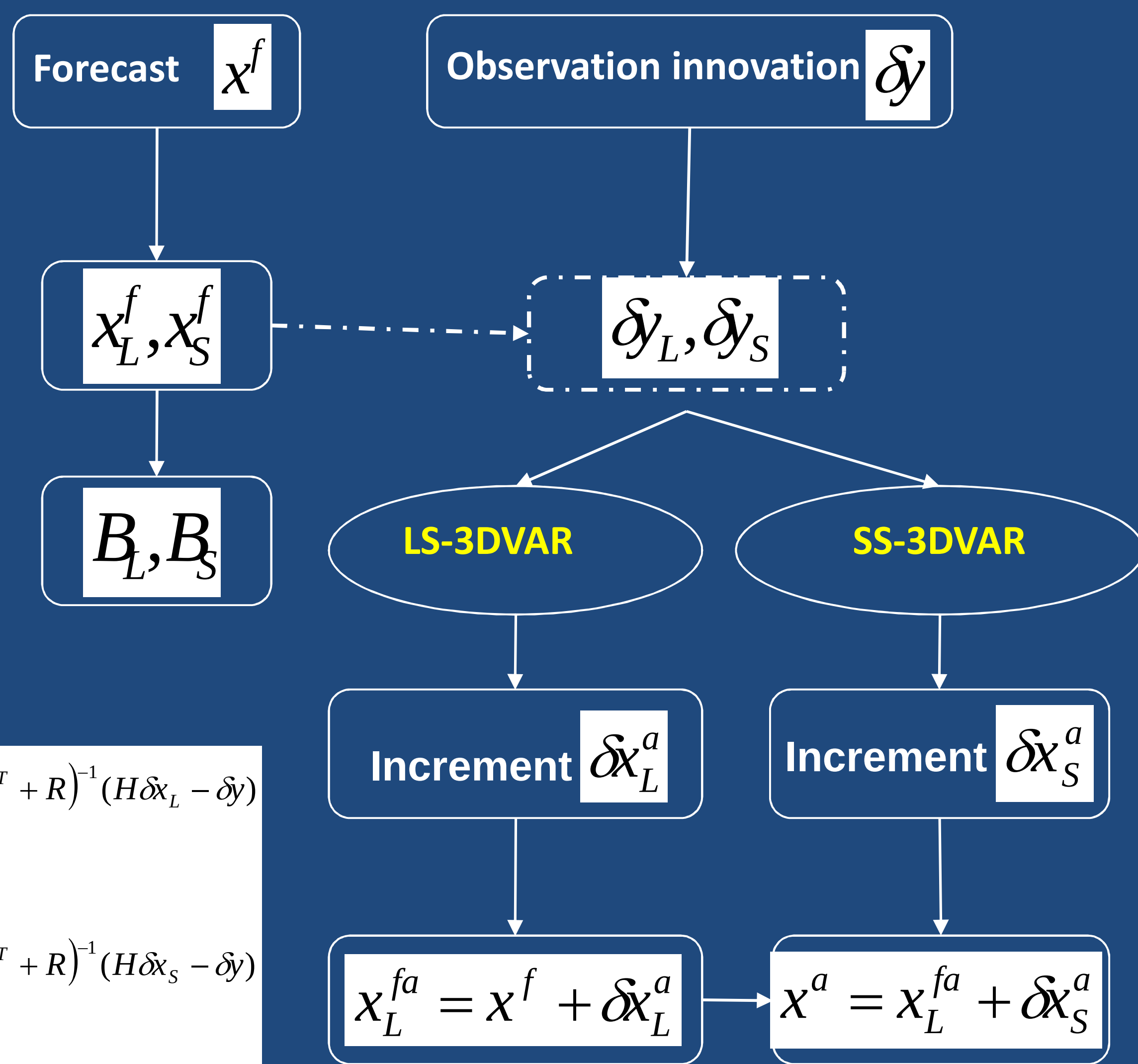
$$B_{ij} = b_e^2 \exp\left[-\frac{(i-j)^2}{2D^2} \right]$$



Analysis Errors



MS-3DVAR Work Flow

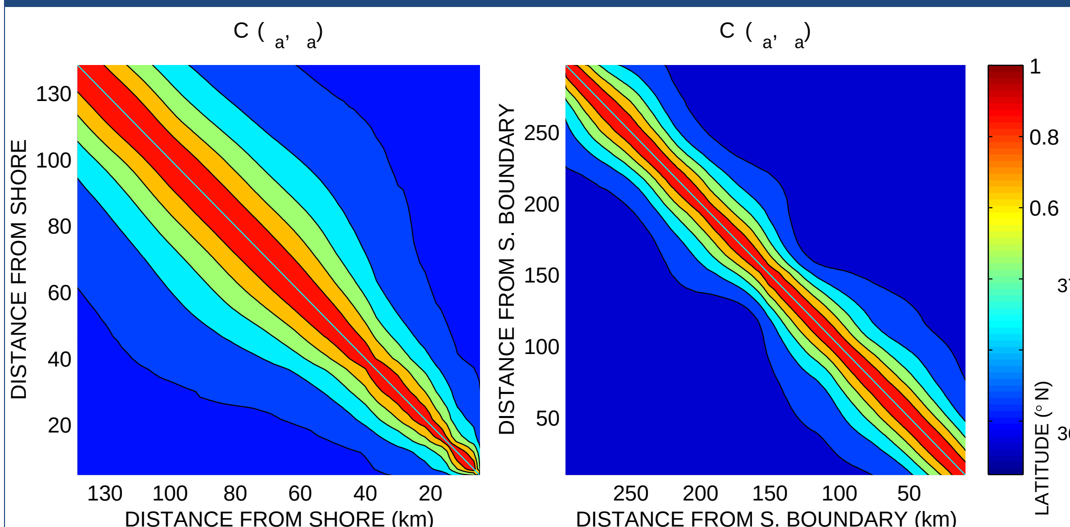


$$\min_{\delta x_L} J = \frac{1}{2}\delta x_L{}^T B_L^{-1}\delta x_L + \frac{1}{2}(H\delta x_L - \delta y)^T\left(HB_S H^T + R\right)^{-1}(H\delta x_L - \delta y) + \frac{1}{2}(H\delta x_L - \delta y_L^h)^T R_L^{-1}(H\delta x_L - \delta y_L^h)$$

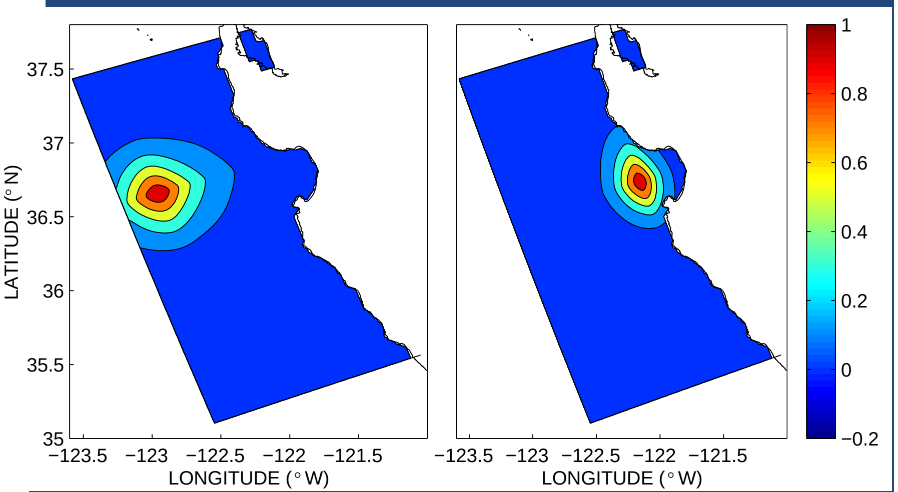
$$\min_{\delta x_S} J = \frac{1}{2}\delta x_S{}^T B_S^{-1}\delta x_S + \frac{1}{2}(H\delta x_S - \delta y)^T\left(HB_L H^T + R\right)^{-1}(H\delta x_S - \delta y) + \frac{1}{2}(H\delta x_S - \delta y_S^h)^T R_S^{-1}(H\delta x_S - \delta y_S^h)$$

Kronecker Product Formulation of 3D Error Correlations

"NMC" Method: 48h-24h Forecast



$$B = \Sigma C \Sigma$$

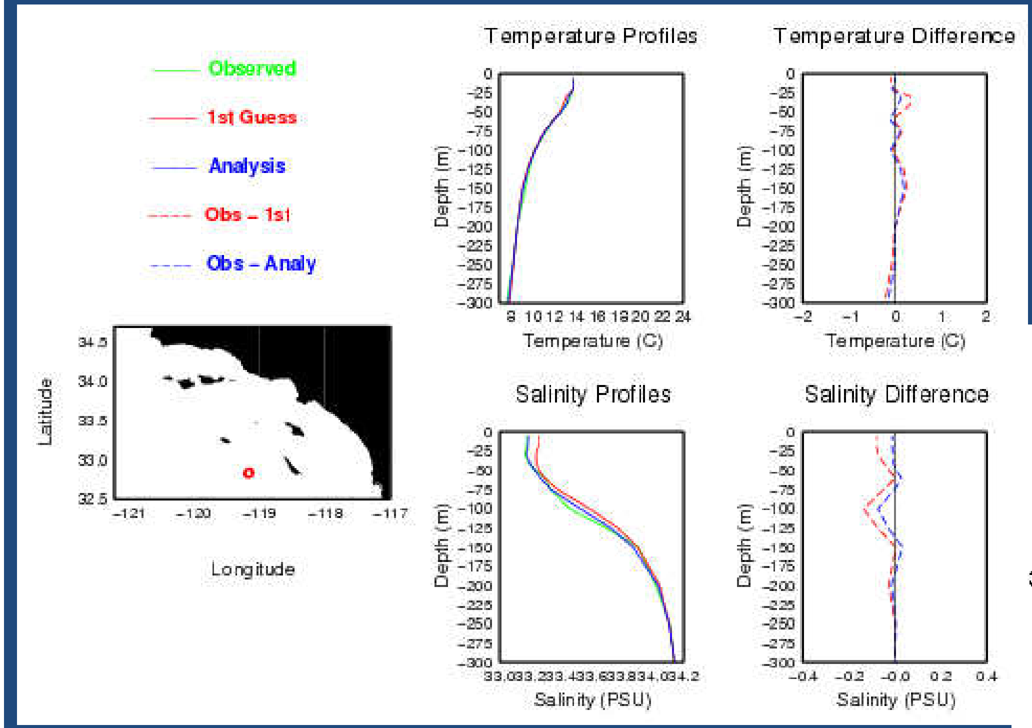


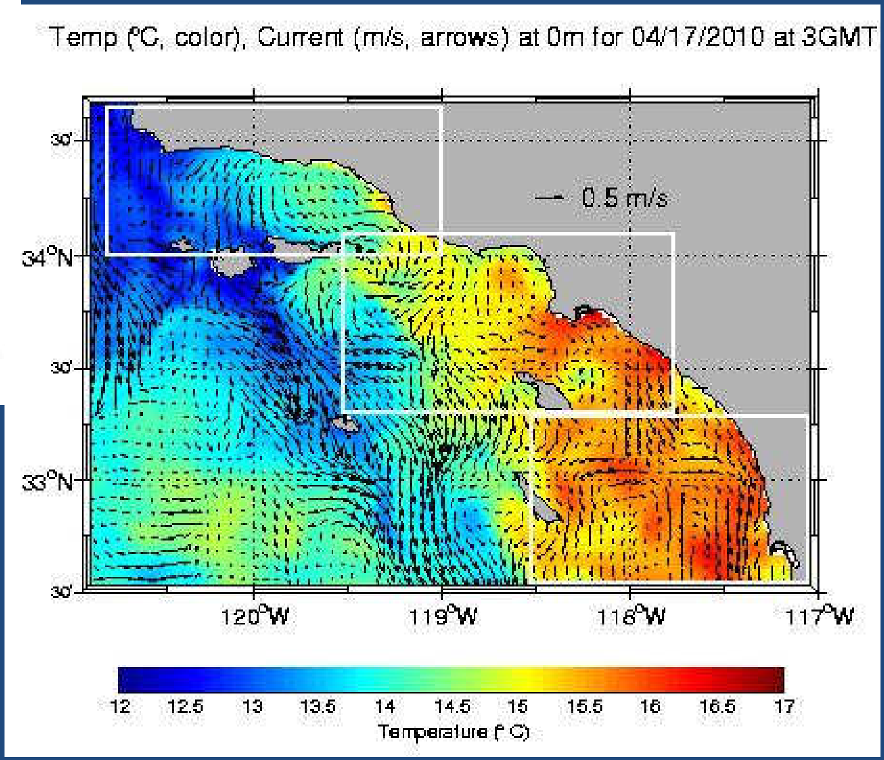
$$C^{\xi\eta\kappa} = C^{\xi\kappa} \otimes C^{\eta}$$

$$C^{\xi\eta\kappa} = G^{\xi\kappa}\left(G^{\xi\kappa}\right)^T \otimes G^{\eta}\left(G^{\eta}\right)^T$$

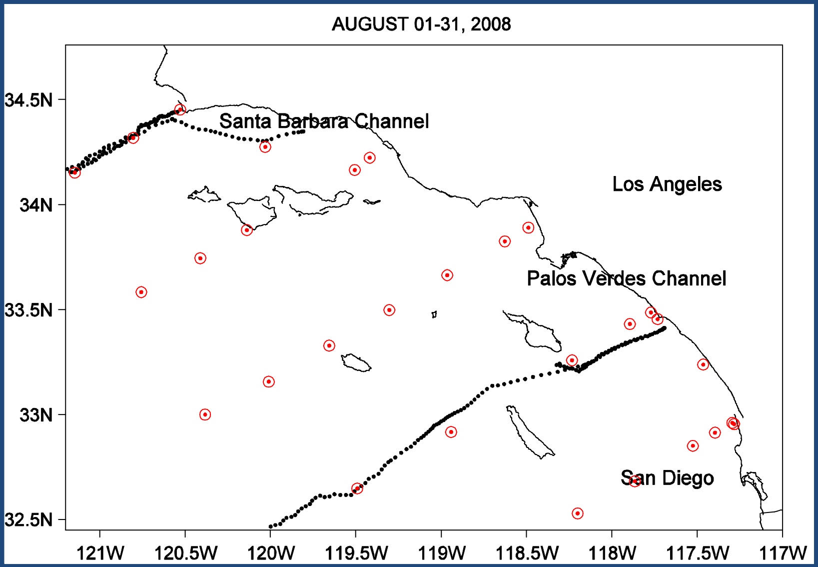
$$= \left(G^{\xi\kappa} \otimes G^{\eta}\right)\left(G^{\xi\kappa} \otimes G^{\eta}\right)^T$$

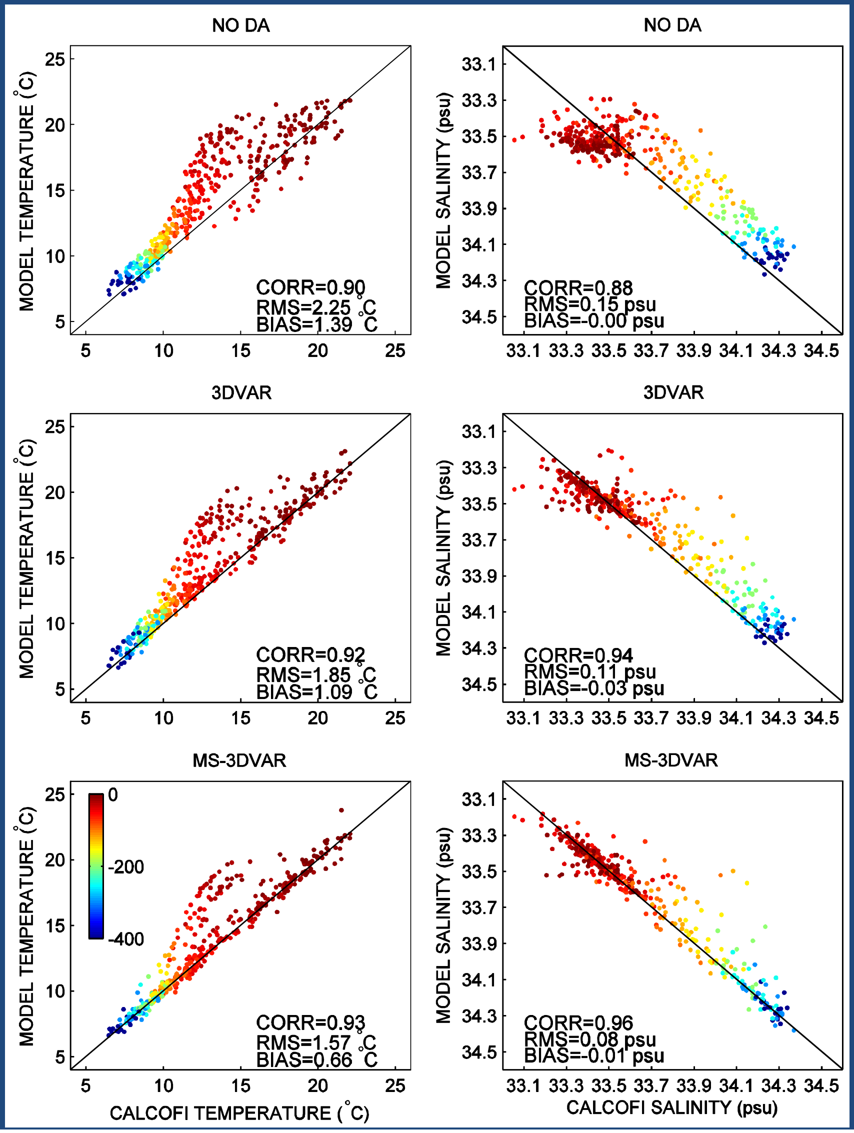
Improved Performance with SCCOOS





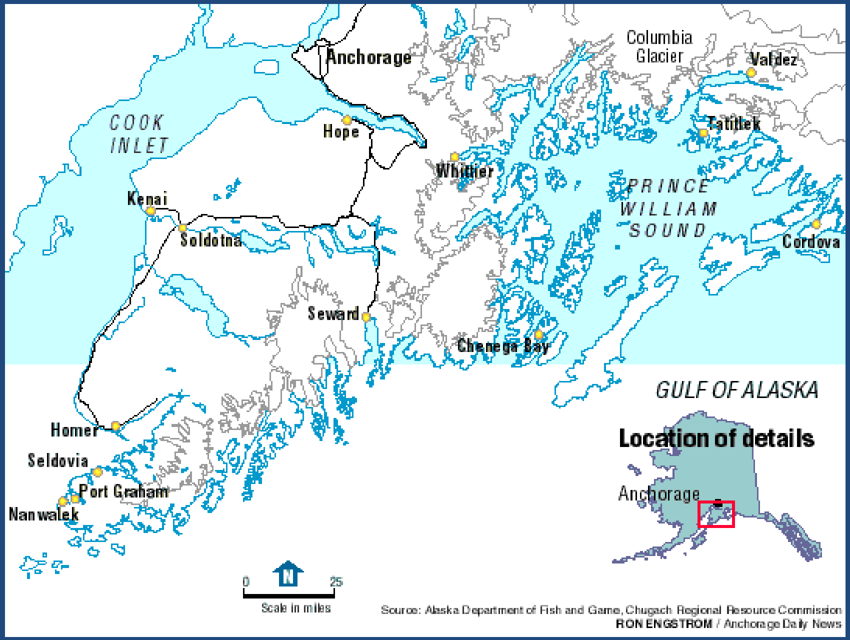
MS-3DVAR Performance





1989 Exxon Valdez Supertanker Oil Spill in the Prince William Sound

Prince William Sound



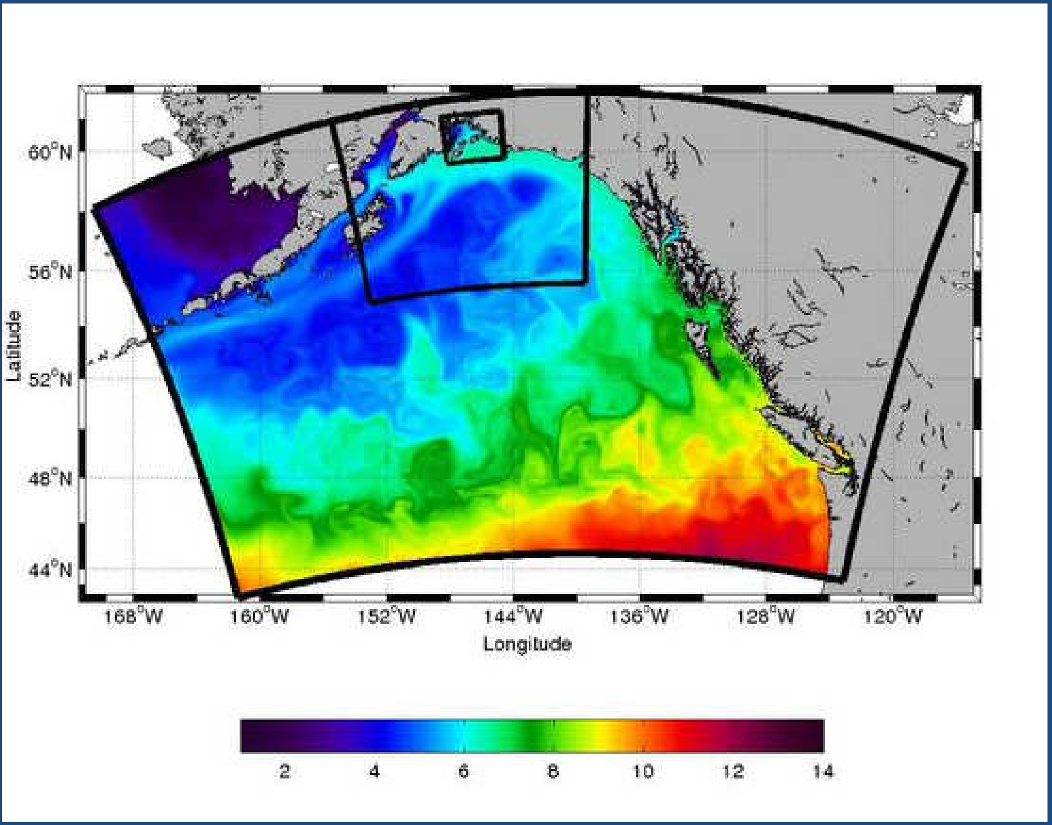
Struggling sea lion during the tragedy days

Blind sea lion present day

Field Experiment 2009 Prediction of Drifter Trajectories in the Prince William Sound

Oil Spill: 1989 Exxon Tanker Wreck , Prince William Sound, Alaska

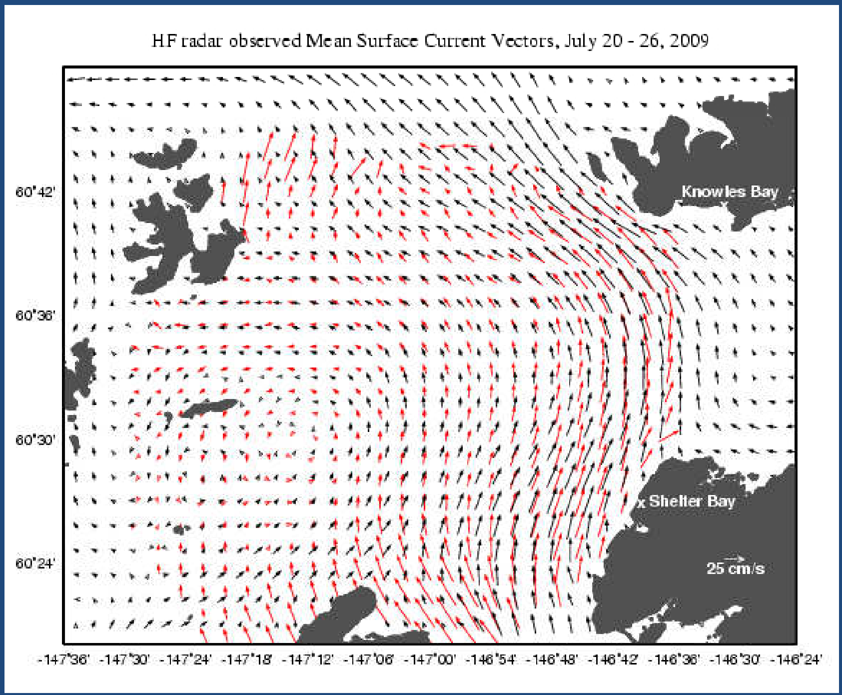
L0 10km
L1 3.6km
L2 1.2km

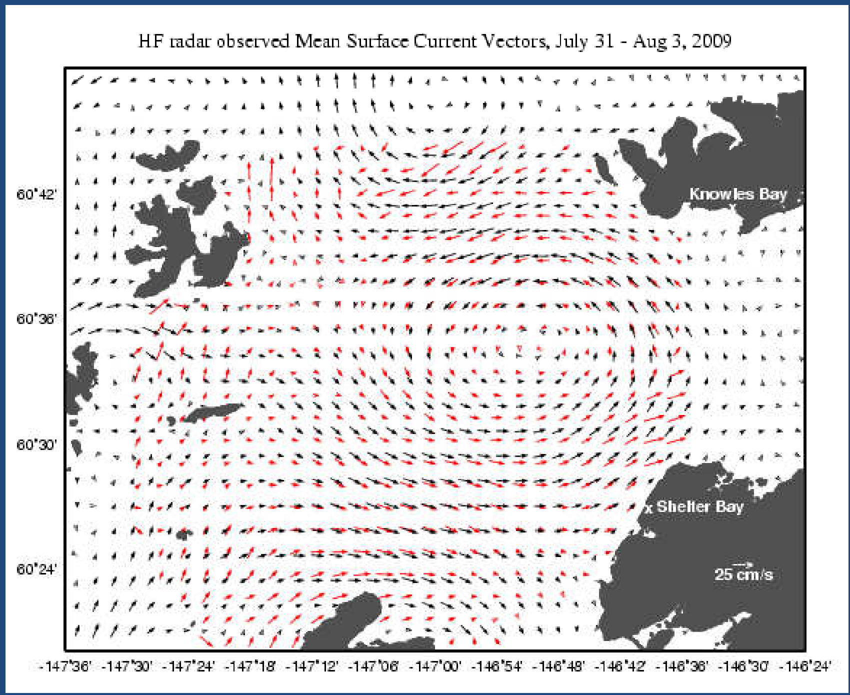


Effective Assimilation of High Frequency Radar High Resolution Velocities during Field Experiment 2009

Surface Currents

HF radar observed (Red), ROMS (Black)





(Schoch and Chao, 2010, EOS)

Summary

- A multi-scale 3DVAR scheme with partitioned cost functions was developed
- MS-3dVAR used multi-decorrelation length scales to construct background error covariance
- Effectiveness of the assimilation of both sparse and high resolution observations was improved
    - Observation oriented covariance
    - Reduced representativeness errors