Modelling convective scale background error covariances using normal modes

Ruth Petrie[1] and Ross Bannister[1]

[1]Deparment of Meteorology
University of Reading

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Modelling the **B**-matrix

The **B** matrix can be modelled via a Control Variable transform (CVT)

$$\delta \mathbf{x} = \mathbf{U}\delta\boldsymbol{\chi}. \tag{1}$$

Substituting this into the incremental background term :

$$J_{\mathrm{b}}(\delta\boldsymbol{\chi}) = \delta\boldsymbol{\chi}^{\mathrm{T}}\mathbf{U}^{\mathrm{T}}\mathbf{B}^{-1}\mathbf{U}\delta\boldsymbol{\chi}, \tag{2}$$

then by choosing U such that:

$$\mathbf{U}^{\mathrm{T}}\mathbf{B}^{-1}\mathbf{U} = \mathbf{I}, \tag{3}$$

results in a simplified background term:

$$J_{\mathrm{b}}(\delta\boldsymbol{\chi}) = \delta\boldsymbol{\chi}^{\mathrm{T}}\delta\boldsymbol{\chi}. \tag{4}$$

Implied B matrix is

$$\mathbf{B}^{\mathrm{i}} = \mathbf{U}\mathbf{U}^{\mathrm{T}} \tag{5}$$

How **B** is represented in reality

Typical structure of U

- Parameter transform decorrelates multivariate relationships.
- Spatial transforms decorrelates univariate relationships.
- Variance scaling ensures **B** is the identity in control space.

Multivariate aspects

- Typically transform to variables which are assumed to be uncorrelated.
- Assume errors in balanced variables are uncorrelated from errors in unbalanced variables
- A mass-wind relationship is used to described balanced flow.

CVTs at the convective scale

At the convective scale:

- the Rossby number is not small and the geostrophic relationship may not be valid.
- the flow is non-hydrostatic and acoustic modes are present.

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An alternative is to use the normal modes of the linearized equations.

- NMs are independent by definition.
- NMs have been used to describe forecast error covariances in the tropics. (Žagar et al, 2004)

Model equations

Model equations

Starting from the standard Euler equations:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \frac{1}{\rho} \nabla p + g\mathbf{k} + \mathbf{f} \times \mathbf{u} = 0, \tag{6a}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \tag{6b}$$

$$\frac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta = 0, \tag{6c}$$

$$p = \rho R \left(\frac{p}{p_{00}} \right)^{\kappa} \theta. \tag{6d}$$

Assume:

- laterally periodic,
- no slip, and rigid lid for the vertical boundary conditions,
- homogeneity in the y-direction,
- f-plane,
- $\phi = \phi_0(z) + \phi'(x, z)$ and $\theta = \theta_{\mathrm{R}} + \theta_0(z) + \theta'(x, z)$.

Approximations

- Impose basic state satisfies hydrostatic balance and the equation of state:

$$\frac{\partial p_0}{\partial z} = -\rho_0 g, \qquad p_0 = \rho_0 R \left(\frac{p_0}{p_{00}}\right)^{\kappa} (\theta_{\mathrm{R}} + \theta_0). \tag{7}$$

- Define the Brunt Väisälä frequency:

$$N^2 = \frac{g}{\theta_{\mathrm{R}}} \frac{d\theta_0}{dz}. \tag{8}$$

- Define buoyancy:

$$b = b_0(z) + b' = \frac{g}{\theta_{\mathrm{R}}} \left(\theta_0(z) + \theta'\right). \tag{9}$$

- Make the Boussinesq approximation (i.e. neglect density perturbations except when multiplied by g).

Simplifications

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- Let N^2 be a tuneable parameter A^2.
- Multiply all advective terms by B (another tuneable parameter).
- Approximate:

$$\frac{\theta'}{\theta_R} = -\frac{\rho'}{\rho_0}. \tag{10}$$

- Adopt simplified equation of state:

$$p = C\rho \qquad \text{where} \qquad C \text{ is a constant.} \tag{11}$$

- Scale density: $\rho = \rho_0(z) + \rho'$

$$(1) \text{ by } \rho_0(z) \rightarrow \tilde{\rho} = 1 + \tilde{\rho}'$$

$$(2) \text{ by } C \rightarrow \tilde{p} = C + \tilde{p}'$$

drop the $\sim$ notation.

Toy model equations

$$\frac{\partial u}{\partial t} + B\mathbf{u}\cdot\nabla u + \frac{\partial p'}{\partial x} - fv = 0, \quad (12a)$$

$$\frac{\partial v}{\partial t} + B\mathbf{u}\cdot\nabla v + fu = 0, \quad (12b)$$

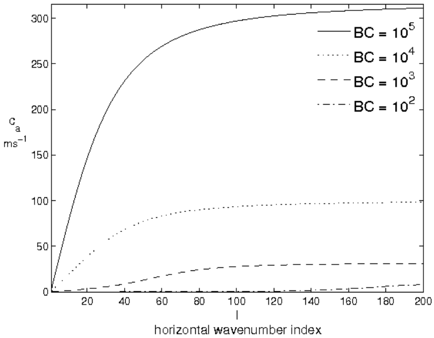
$$\frac{\partial w}{\partial t} + B\mathbf{u}\cdot\nabla w + \frac{\partial p'}{\partial z} - b' = 0, \quad (12c)$$

$$\frac{\partial p'}{\partial t} + B\nabla\cdot(p\mathbf{u}) = 0, \quad (12d)$$

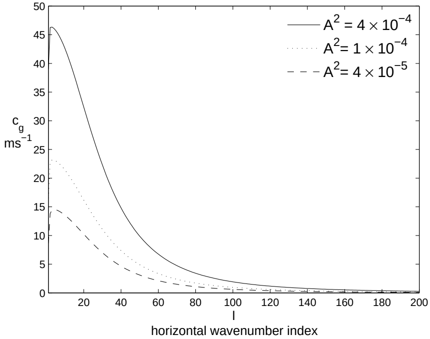
$$\frac{\partial b'}{\partial t} + B\mathbf{u}\cdot\nabla b' + A^2 w = 0. \quad (12e)$$

- Model conserves energy analytically.
- 360 longitudinal points, $\delta x = 1.5\mathrm{km}$.
- 60 vertical levels, $\delta z \sim 260\mathrm{m}$.
- Centered-in-time, forward-backward (Cullen and Davies, 1991).

Linear analysis results



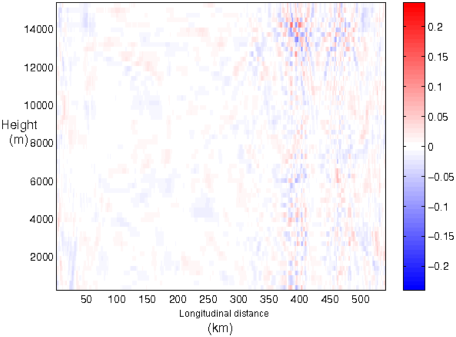
Acoustic wave speed sensitivity to BC; $A^2 = 4 \times 10^{-4}\ \mathrm{s}^{-2}$

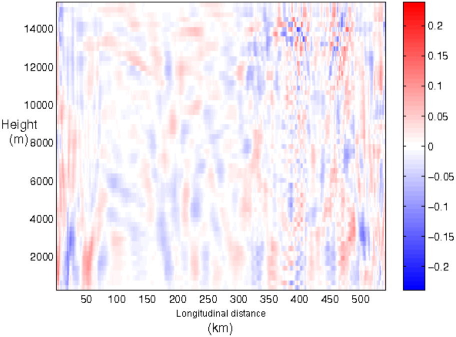


Gravity sensitivity to A^2; $BC = 10^5\ \mathrm{m}^2\mathrm{s}^{-2}$

Convective-like behaviour

w - vertical component of wind

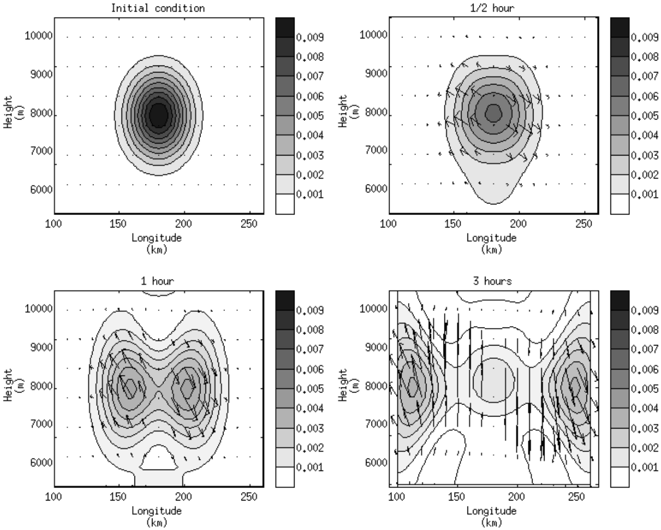




$$A^2 = 4 \times 10^{-4}, \quad B = 10^{-2}, \; C = 10^4$$

$$A^2 = 4 \times 10^{-5}, \quad B = 10^{-2}, \; C = 10^4$$

Geostrophic adjustment



Structure of NMCVT

$$\delta\boldsymbol{\chi} = \mathbf{U}_{\mathbf{I}}^{-1}\mathbf{U}_{\mathrm{E}}^{-1}\mathbf{U}_{\mathrm{S}}^{-1}\mathbf{U}_{\mathrm{V}}^{-1}\mathbf{U}_{\mathrm{M}}^{-1}\mathbf{U}_{\mathrm{H}}^{-1}\delta\mathbf{x} \tag{13}$$

$\mathbf{U}_{\mathrm{H}}^{-1}$ is a horizontal Fourier transform.

$\mathbf{U}_{\mathrm{M}}^{-1}$ is a Helmholtz variable transform.

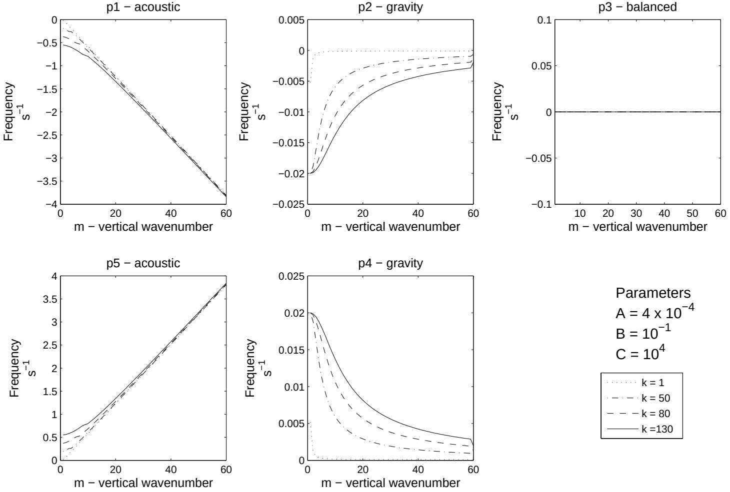
$\mathbf{U}_{\mathrm{V}}^{-1}$ is a vertical Fourier transform.

$\mathbf{U}_{\mathrm{S}}^{-1}$ is a symmetric scaling.

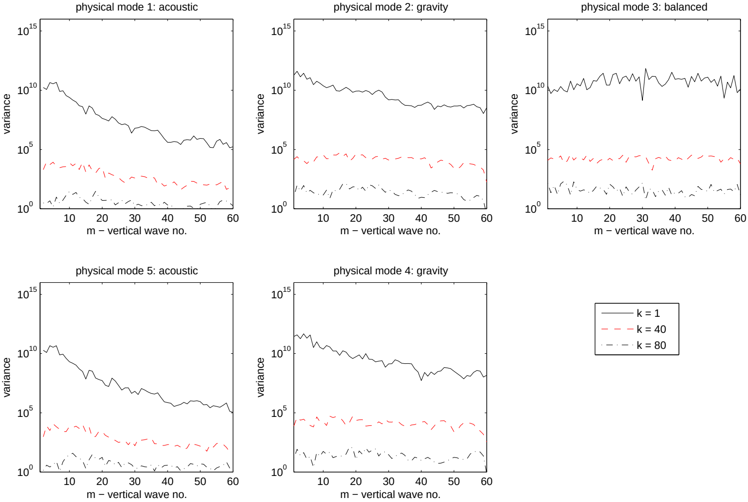
$\mathbf{U}_{\mathrm{E}}^{-1}$ is a projection on to eigenvectors.

$\mathbf{U}_{\mathbf{I}}^{-1}$ is a scaling to ensure unit variance.

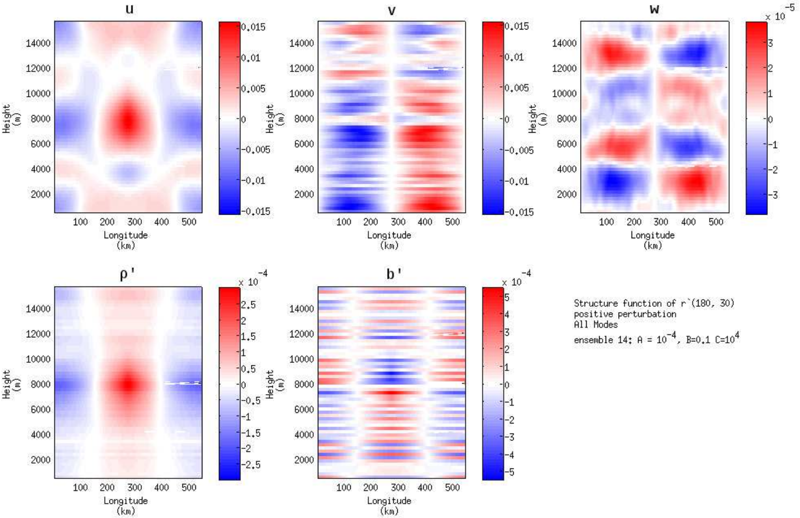
Normal modes



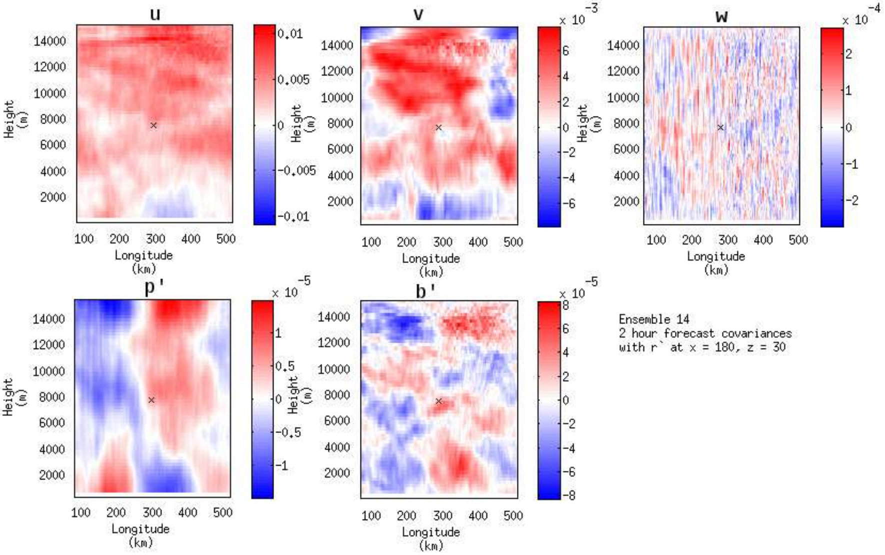
Covariance spectrum



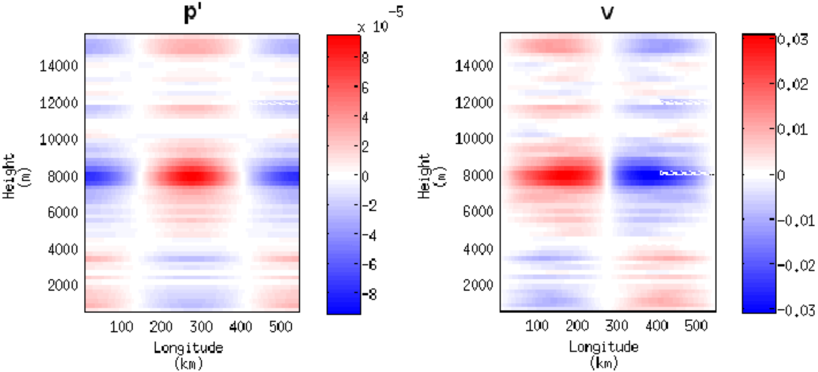
Implied covariances: including all modes



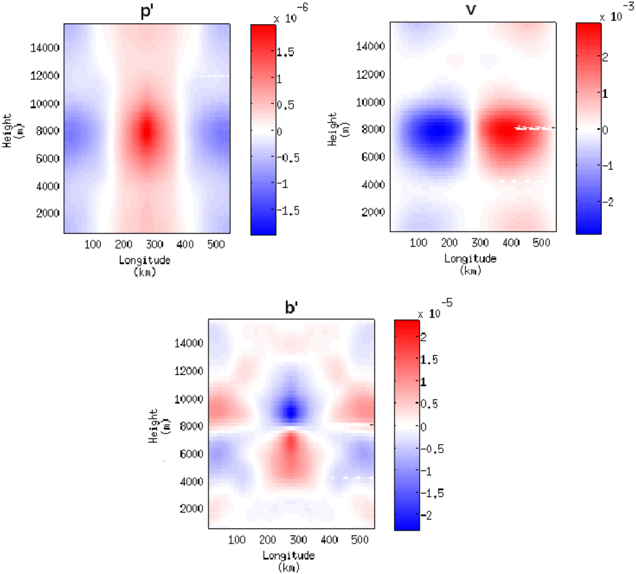
Ensemble derived covariances



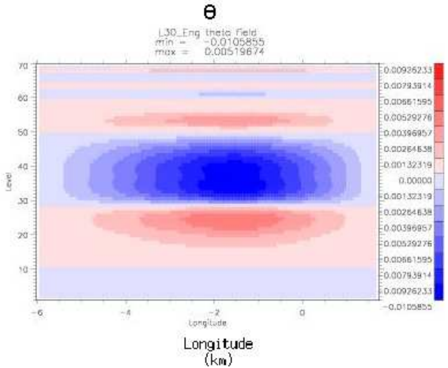
Implied covariances: balanced mode only

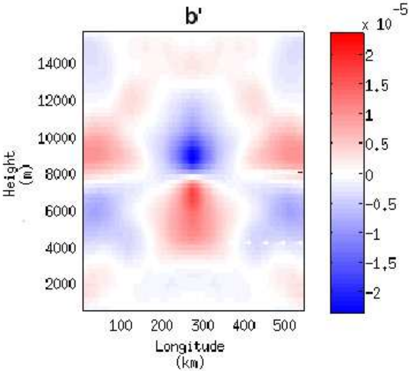


Implied covariances: gravity modes only

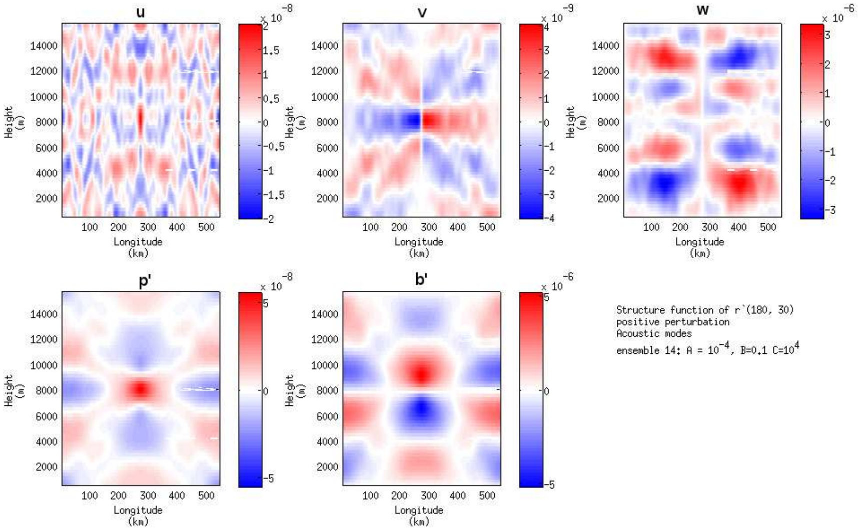


Implied covariances: comparison





Implied covariances: acoustic modes only



Future work

- Investigate the degree to which the linearised normal modes are uncorrelated in the non-linear system.
- Compare the NMCVT with a standard approach.
- Assess the impact of the NMCVT inside an assimilation system.
- Hybrid methods.
- What covariances are appropriate at the convective scale?

Summary

- A non-hydrostatic toy model has been developed as a tool to investigate the convective scale data assimilation problem.
- A normal mode approach to covariance modelling has been adopted.
- Both the model and the covariance model should be a useful tool in further investigating the convective scale data assimilation problem.

Thank you for your attention,
any questions/comments?

Eigenvectors