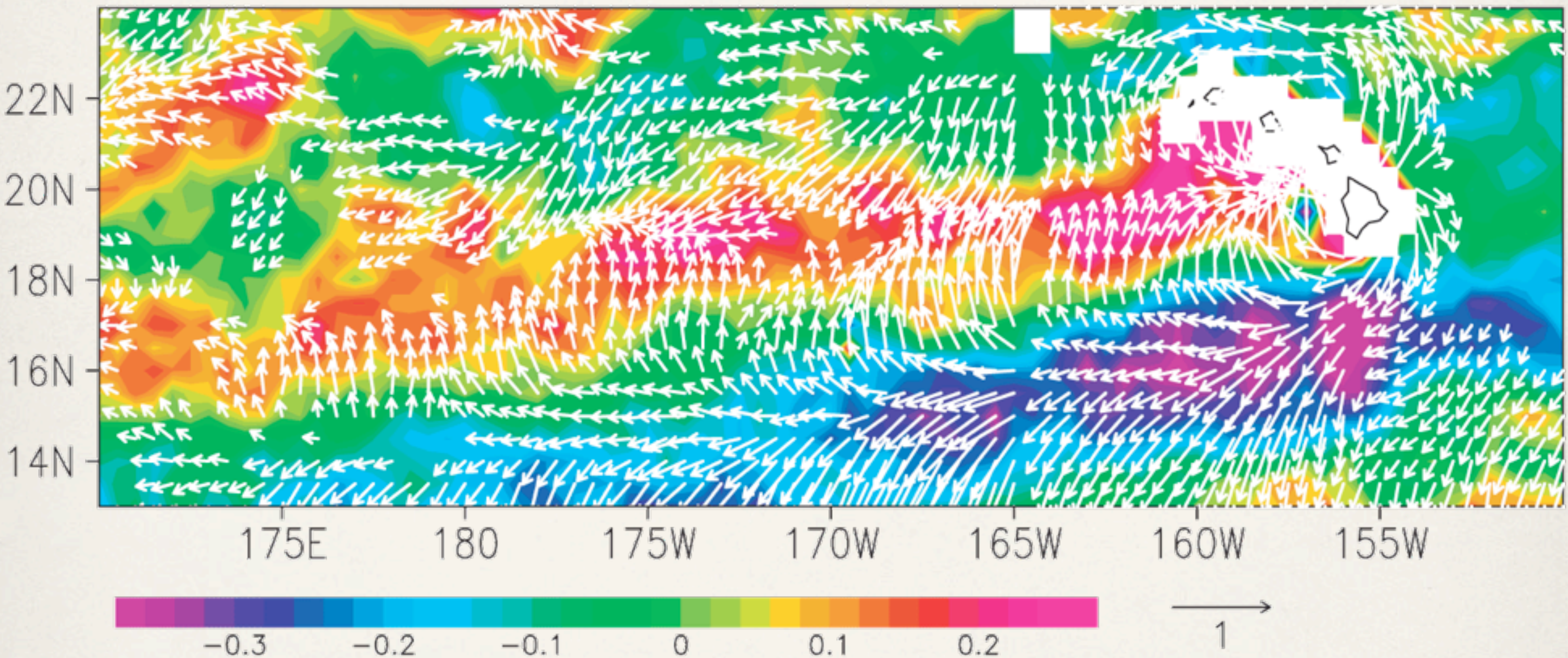
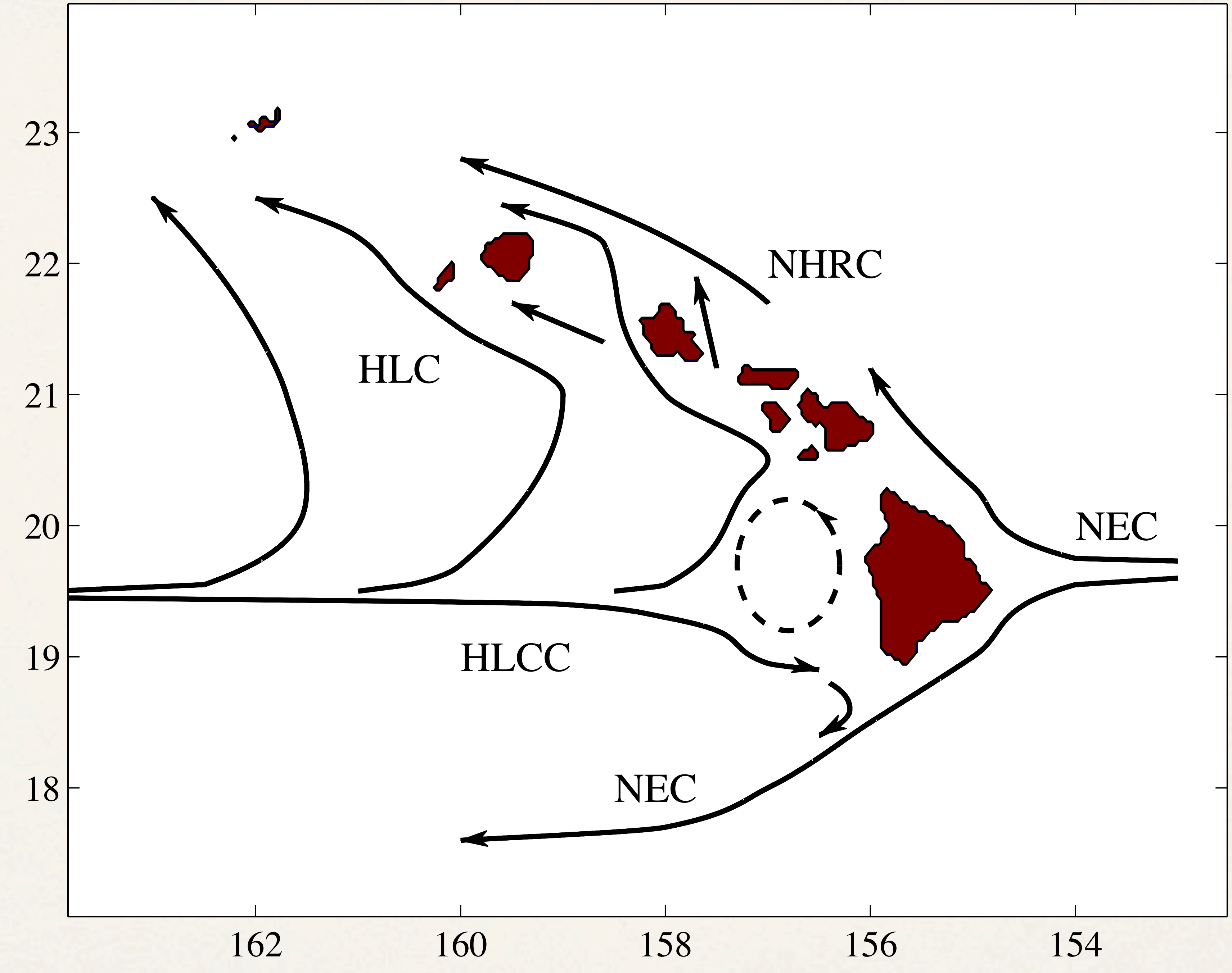
Quantifying the role of observations in ocean state estimation.

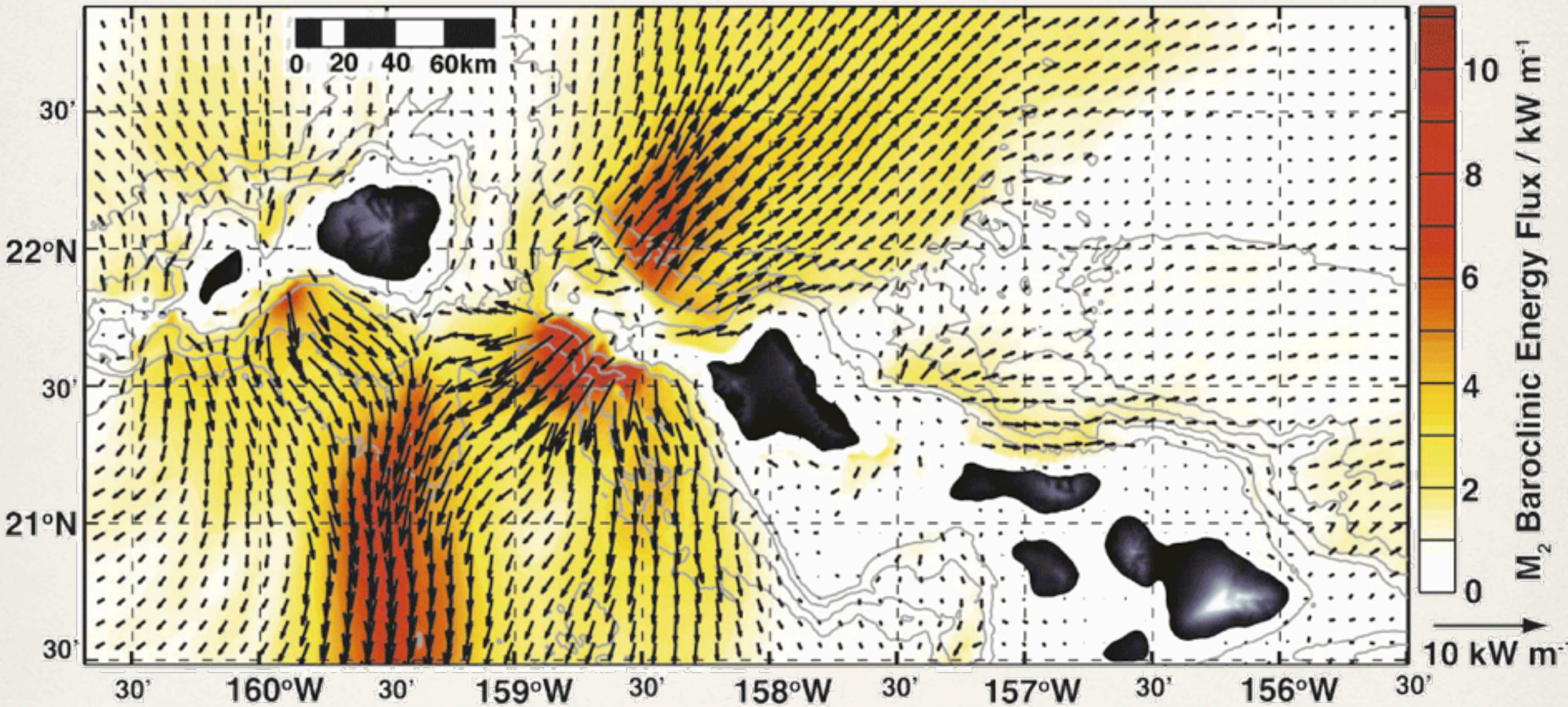
Brian Powell, University of Hawaii



S.-P. Xie, W. Liu, Q. Liu, and M. Nonaka. Far-Reaching Effects of the Hawaiian Islands on the Pacific Ocean-Atmosphere System. Science, 292(5524):2057–2060, 2001.



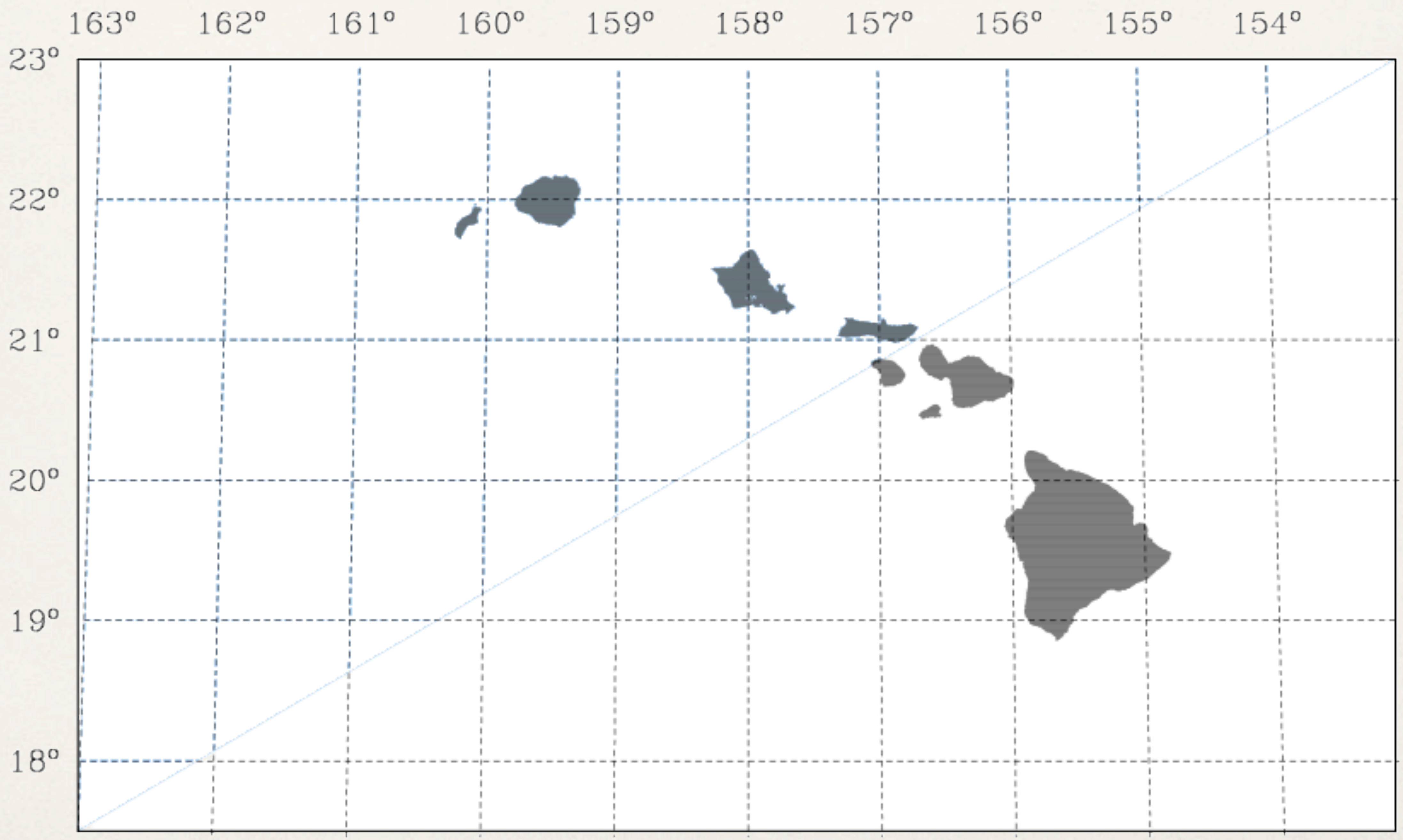
23
22
21
20
19
18
162
160
158
156
154
NHRC
HLC
NEC
HLCC
NEC

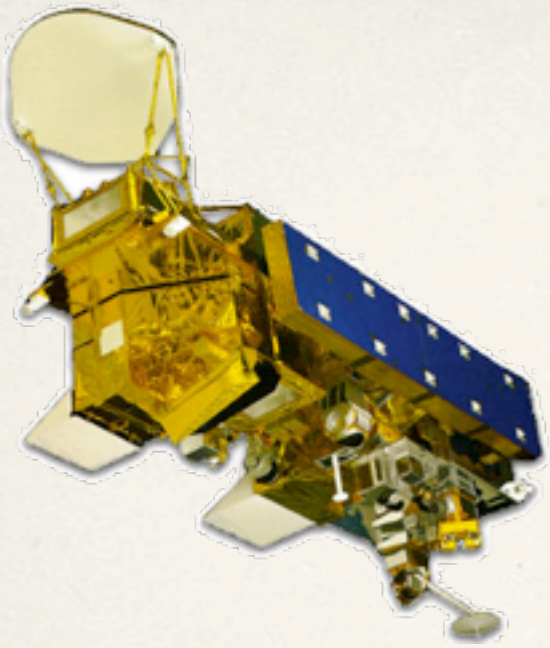


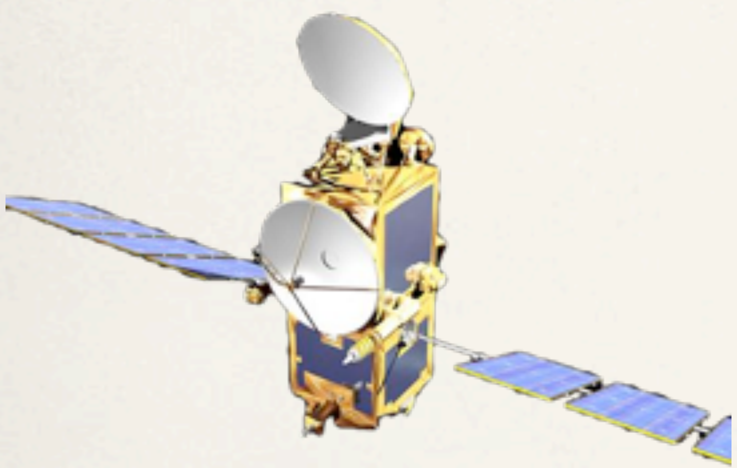
G. S. Carter, M. A. Merrifield, J. M. Becker, K. Katsumata, M. C. Gregg, D. S. Luther, M. D. Levine, T. J. Boyd, and Y. L. Firing. Energetics of M2 Barotropic-to-Baroclinic Tidal Conversion at the Hawaiian Islands. J. Phys. Oceanogr., 38:2,205–2,223, 2008.

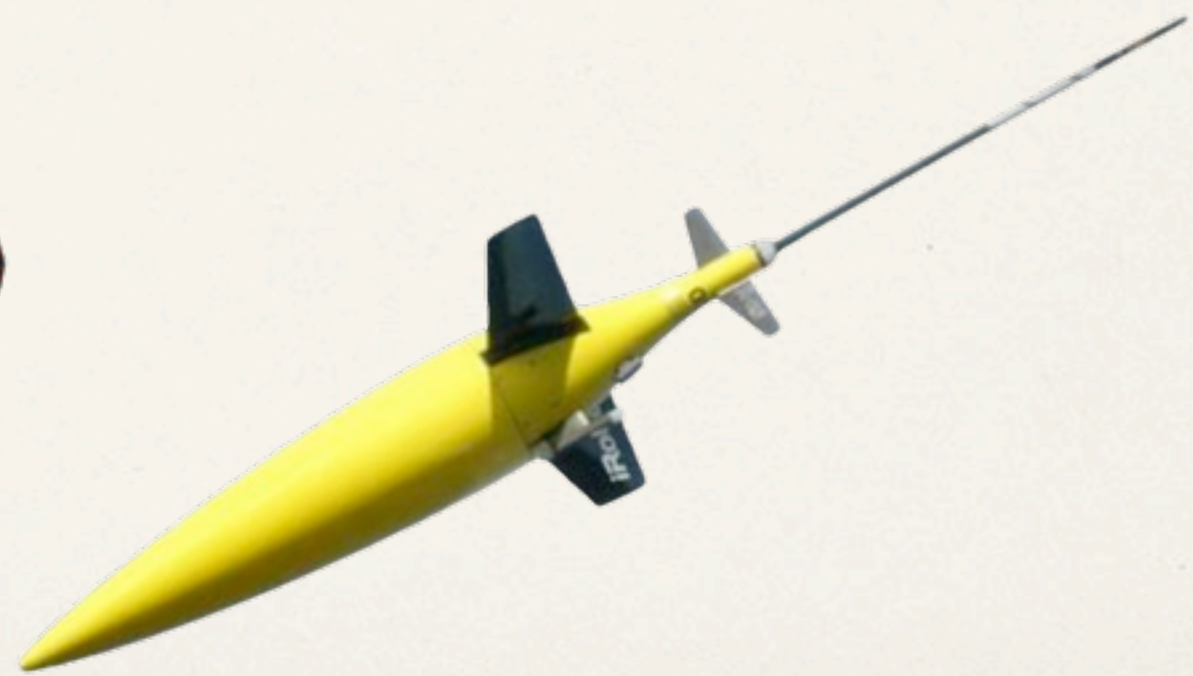
PacIOOS Pacific Islands Ocean Observing System

IN THE SCHOOL OF OCEAN AND EARTH SCIENCE AND TECHNOLOGY AT THE UNIVERSITY OF HAWAI'I AT MĀNOA










ARGO

Observation	Count	Percent
HOT Temperature	4,982	0.02%
HOT Salt	4,982	0.02%
Argo Temp	15,212	0.06%
Argo Salt	15,212	0.06%
Seaglider Temperature	220,266	0.83%
Seaglider Salt	220,266	0.83%
SST	25,201,519	94.50%
SSH	985,731	3.70%
Total	26,668,170	

- Recall, $$\mathbf{x}_a = \mathbf{x}_b + \mathbf{K}\left(\mathbf{y} - \mathbf{H}\mathbf{x}_b\right)$$

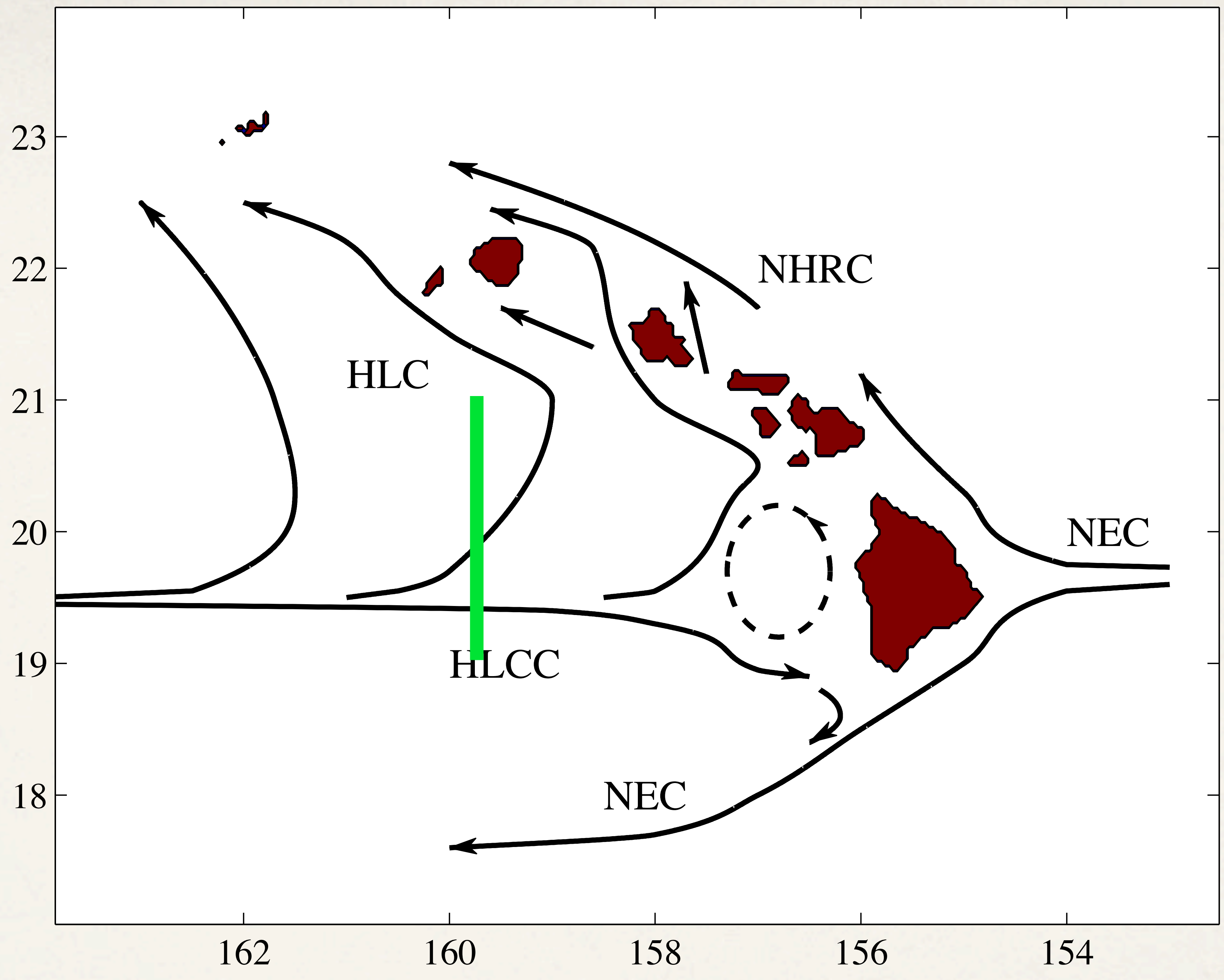
$$\mathbf{K} = \left(\mathbf{G}^T\mathbf{R}^{-1}\mathbf{G} + \mathbf{P}^{-1}\right)^{-1}\mathbf{G}^T\mathbf{R}^{-1}$$

- We have some measure of the ocean, $\mathcal{J}$
- Between the analysis and background, we have:

$$\Delta\mathcal{J} = \mathcal{J}\left(\mathbf{x}_b + \mathbf{K}\left(\mathbf{y} - \mathbf{H}\mathbf{x}_b\right)\right) - \mathcal{J}\left(\mathbf{x}_b\right)$$

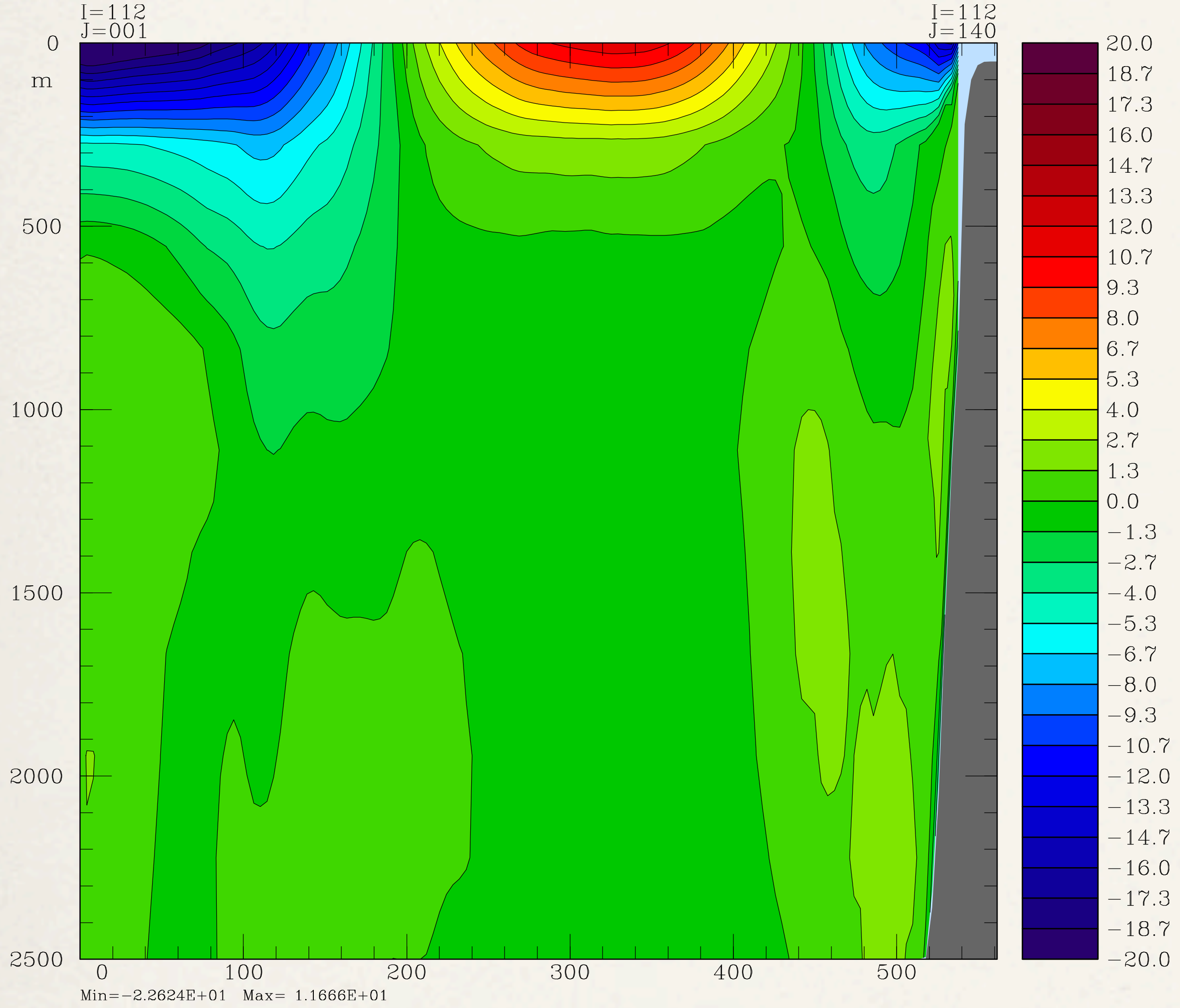
- Following Langland and Baker (2004), Errico (2007), first-order Taylor Expansion:

$$\Delta\mathcal{J} = 2\left(\mathbf{y} - \mathbf{H}\mathbf{x}_b\right)^T\mathbf{K}^T\mathbf{M}_b^T\frac{\partial\mathcal{J}}{\partial\mathbf{x}_b}$$

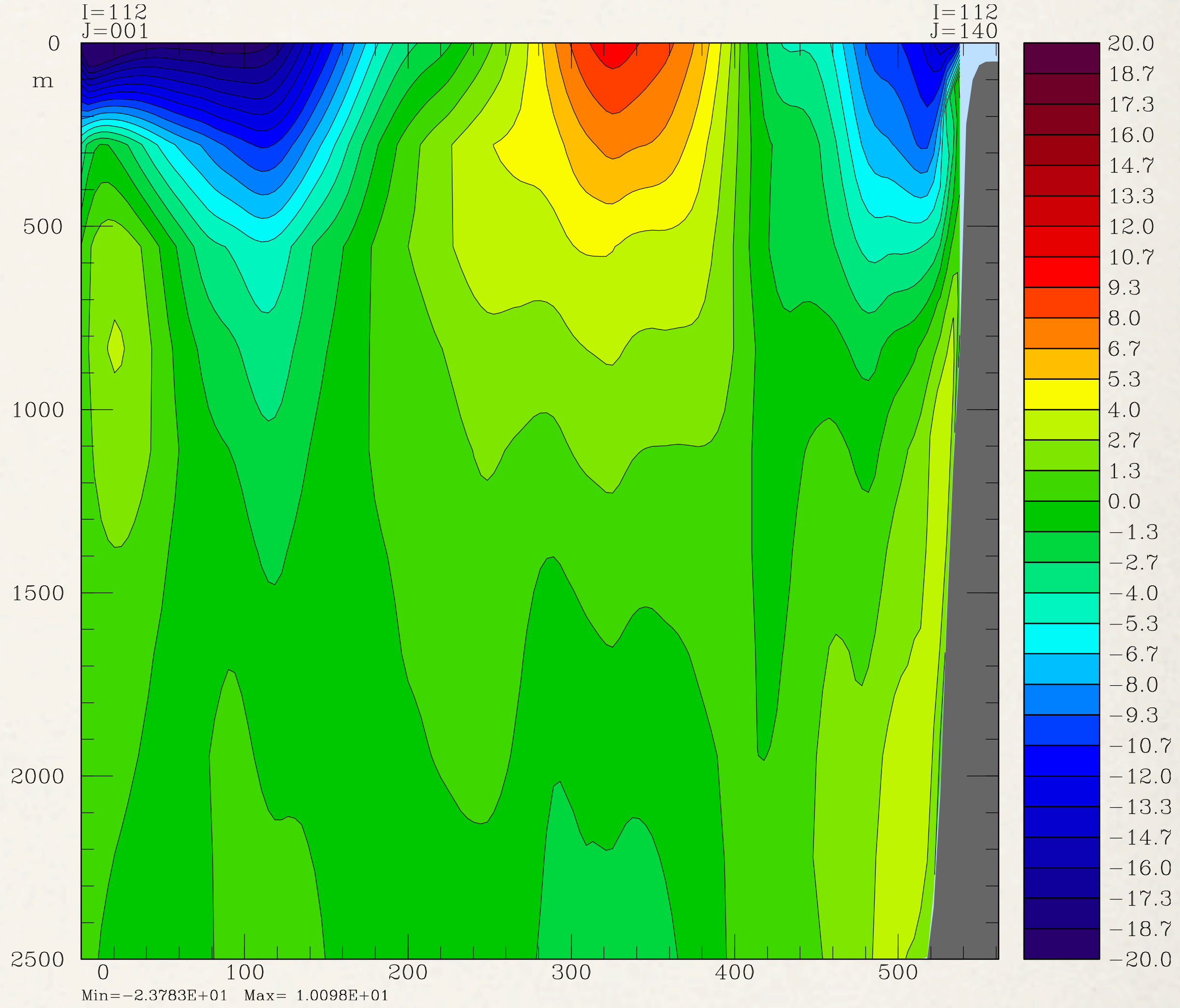


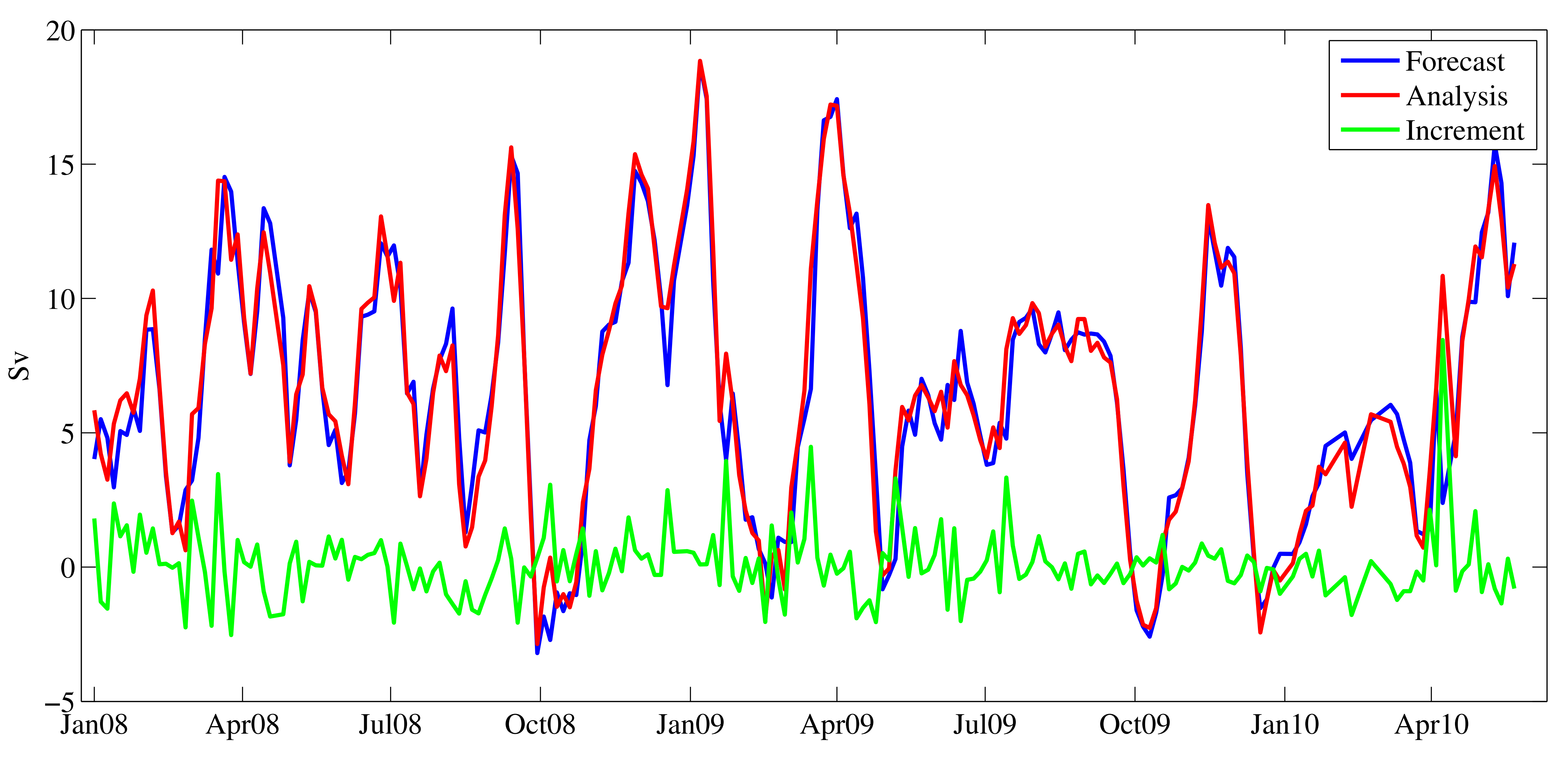
$$\mathcal{J} = \frac{1}{T \cdot 10^6} \int_T \int_S \int_{-Z}^{0} u \Delta y \mathrm{d}z \mathrm{d}s \mathrm{d}t$$

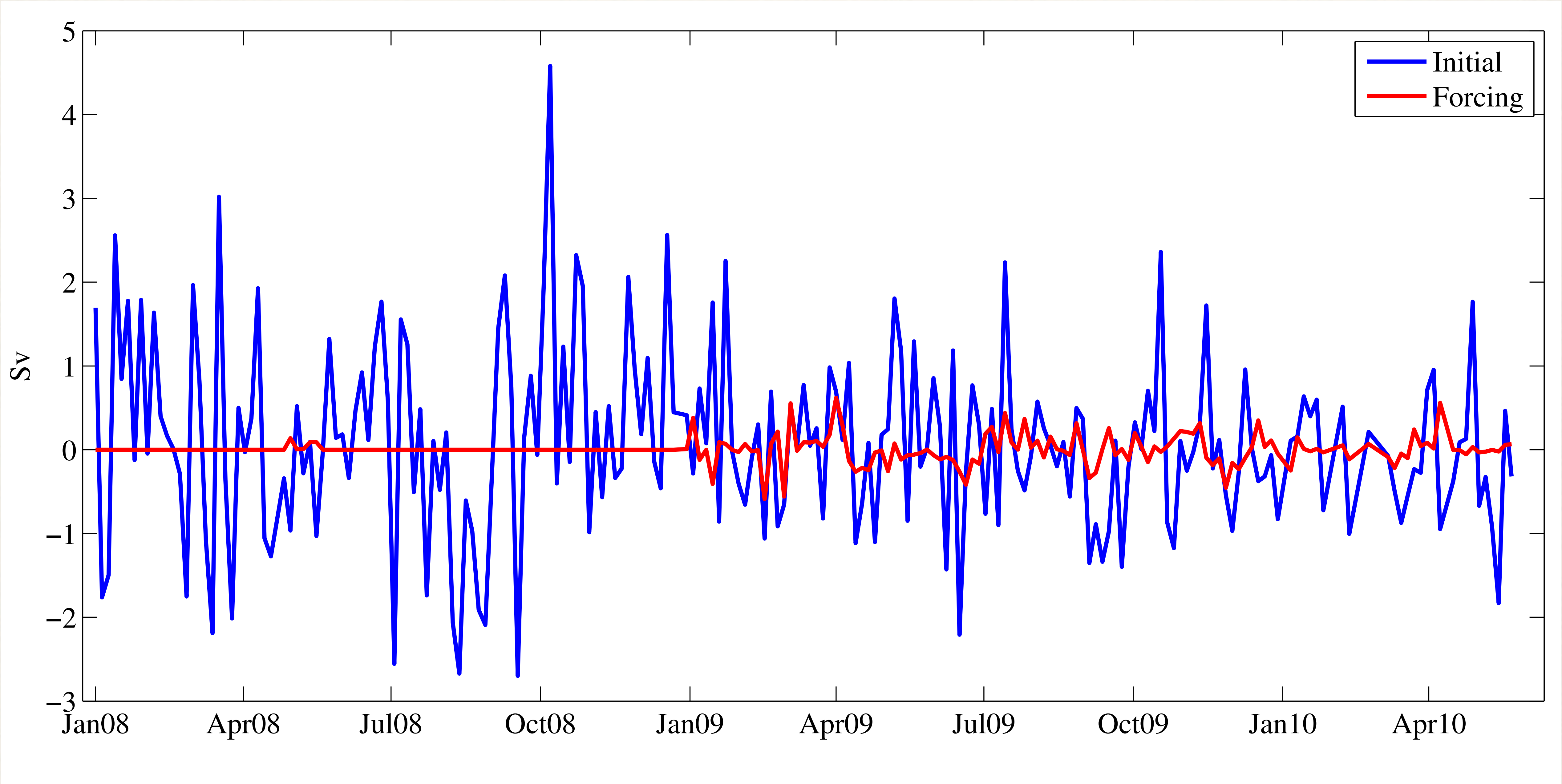
Forecast



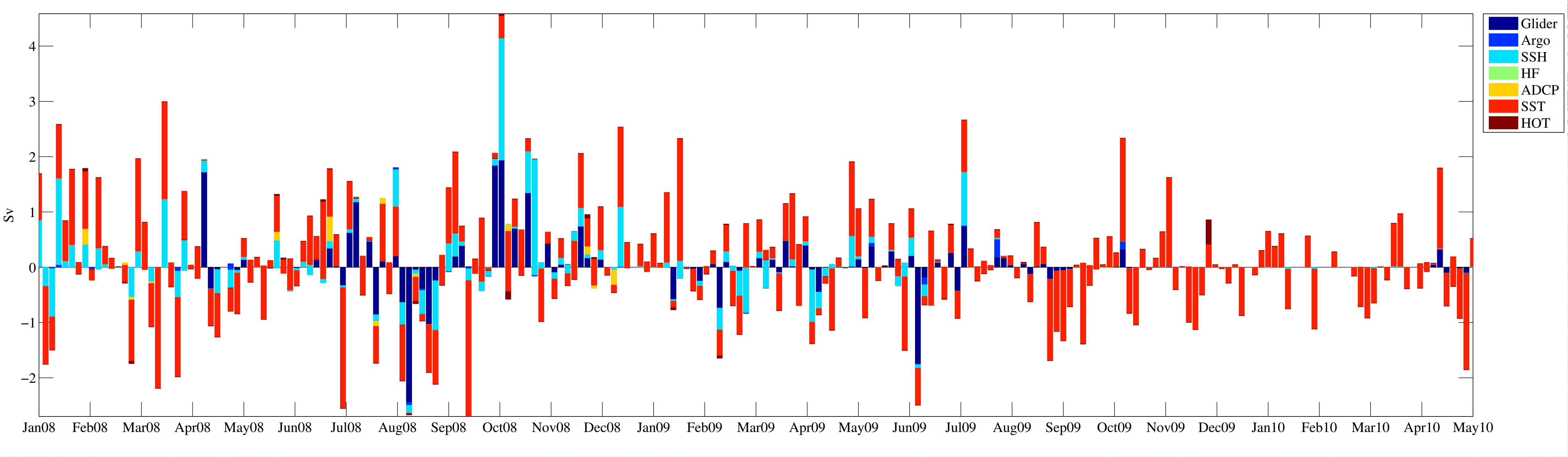
Analysis



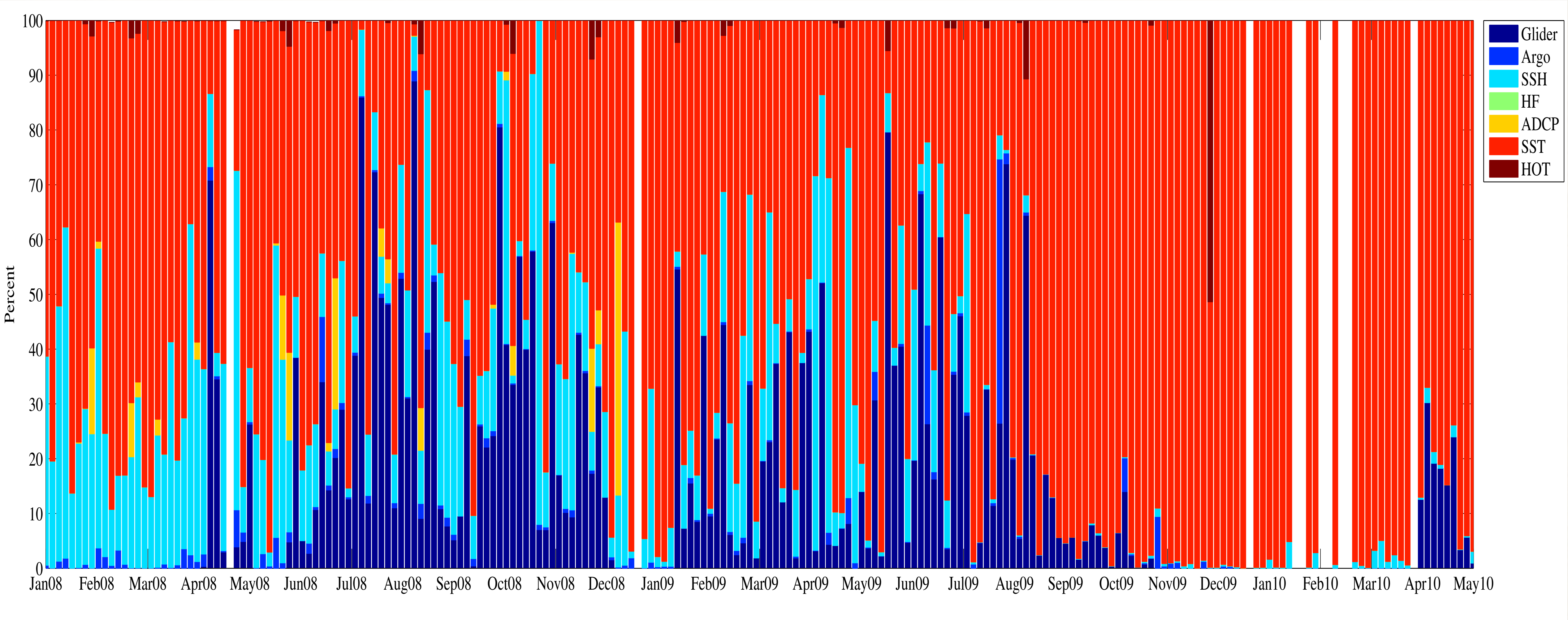

20
15
10
5
0
−5
Sv
Forecast
Analysis
Increment
Jan08
Apr08
Jul08
Oct08
Jan09
Apr09
Jul09
Oct09
Jan10
Apr10



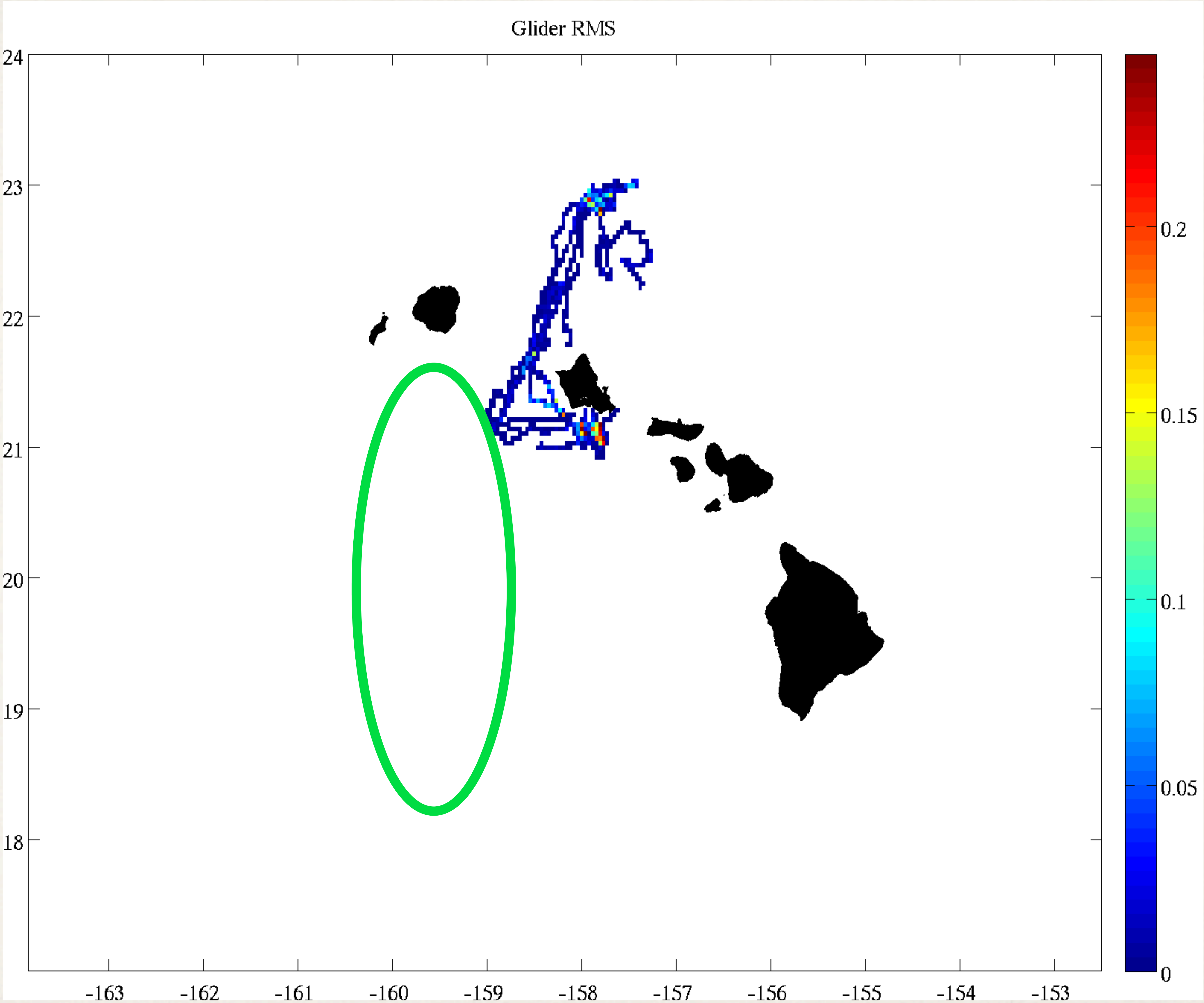
Initial
Forcing
Sv
5
4
3
2
1
0
−1
−2
−3
Jan08
Apr08
Jul08
Oct08
Jan09
Apr09
Jul09
Oct09
Jan10
Apr10



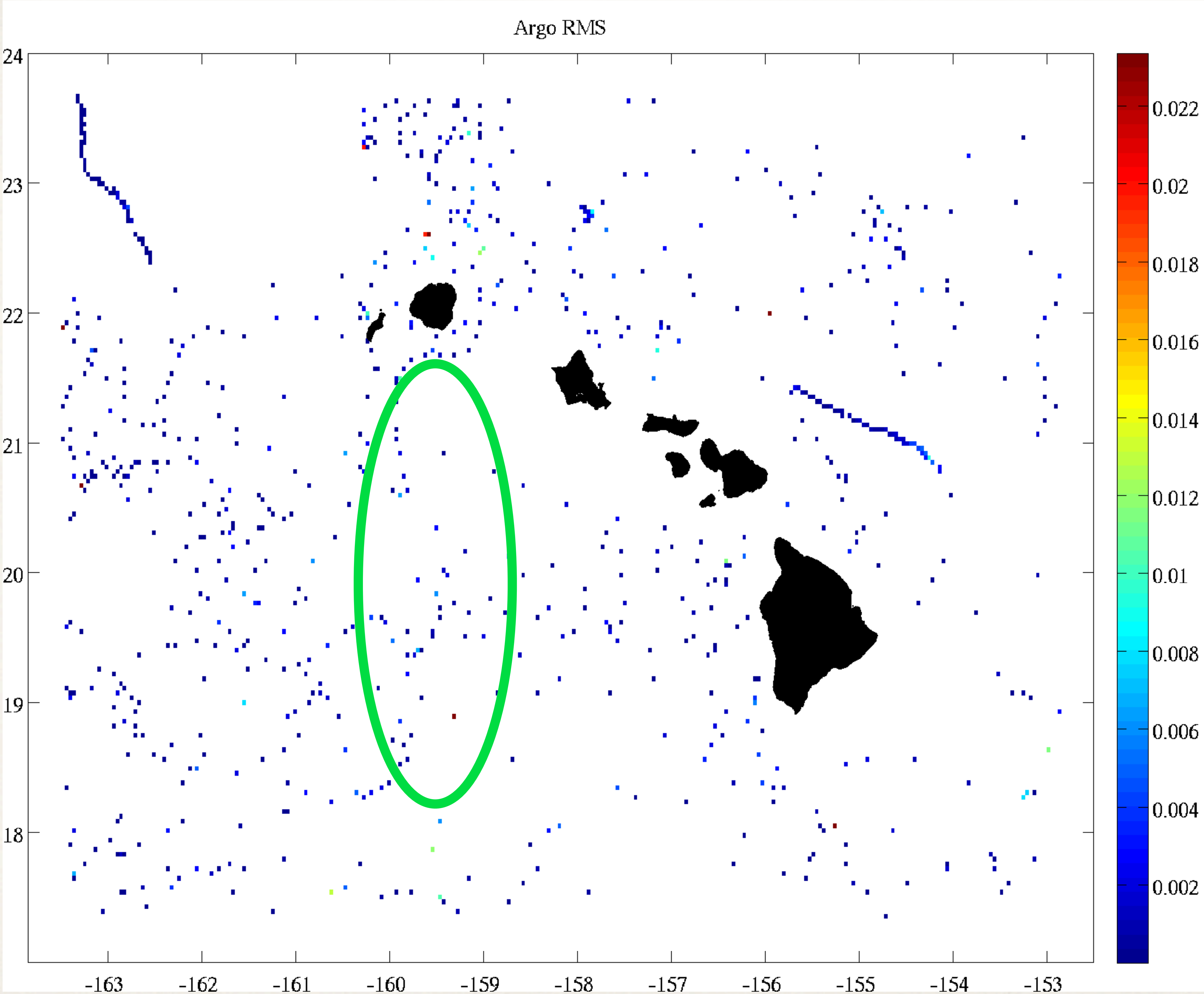
Sv
4
3
2
1
0
−1
−2
Jan08 Feb08 Mar08 Apr08 May08 Jun08 Jul08 Aug08 Sep08 Oct08 Nov08 Dec08 Jan09 Feb09 Mar09 Apr09 May09 Jun09 Jul09 Aug09 Sep09 Oct09 Nov09 Dec09 Jan10 Feb10 Mar10 Apr10 May10
Glider
Argo
SSH
HF
ADCP
SST
HOT



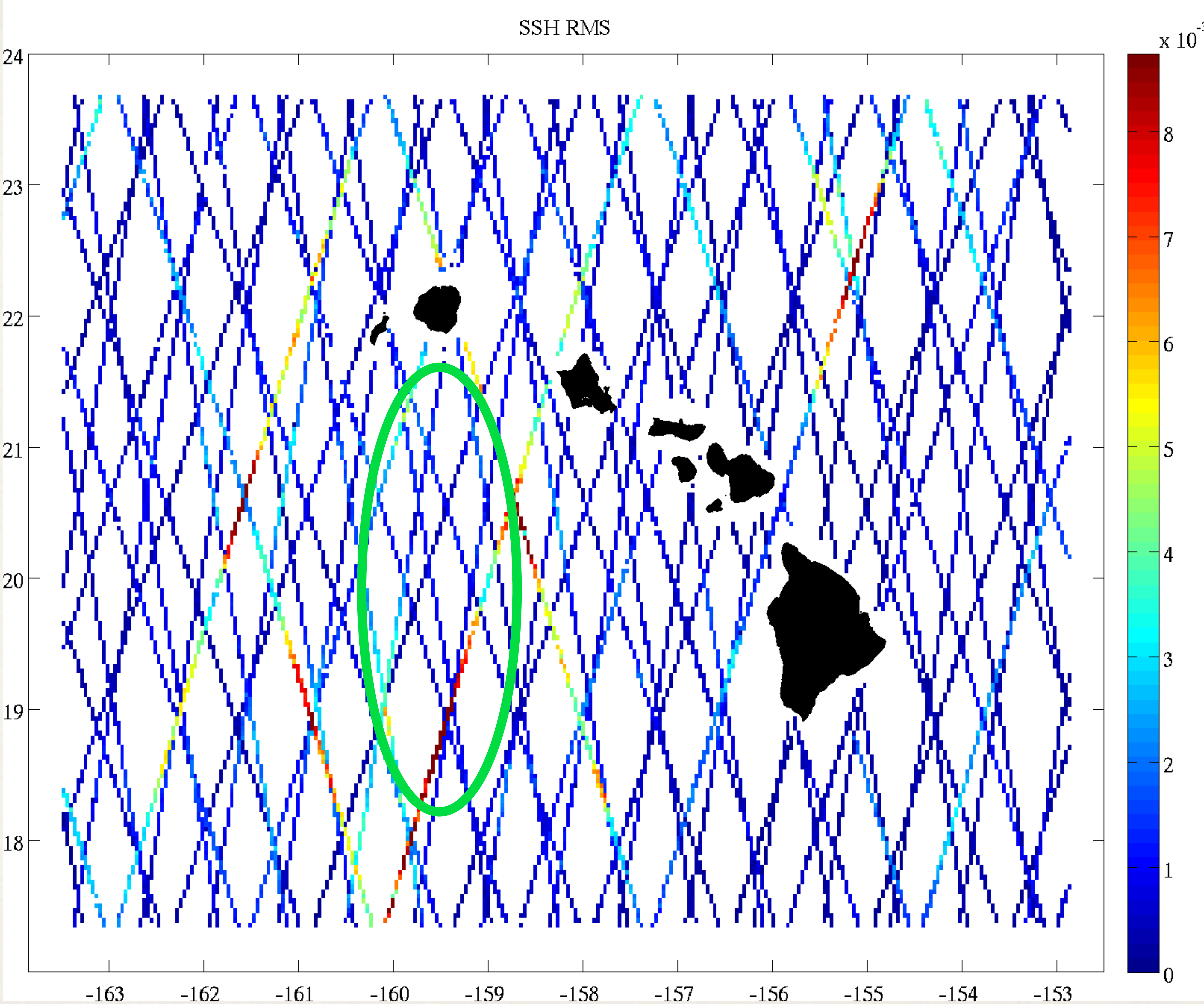
Percent
100
90
80
70
60
50
40
30
20
10
0
Jan08 Feb08 Mar08 Apr08 May08 Jun08 Jul08 Aug08 Sep08 Oct08 Nov08 Dec08 Jan09 Feb09 Mar09 Apr09 May09 Jun09 Jul09 Aug09 Sep09 Oct09 Nov09 Dec09 Jan10 Feb10 Mar10 Apr10 May10
Glider
Argo
SSH
HF
ADCP
SST
HOT

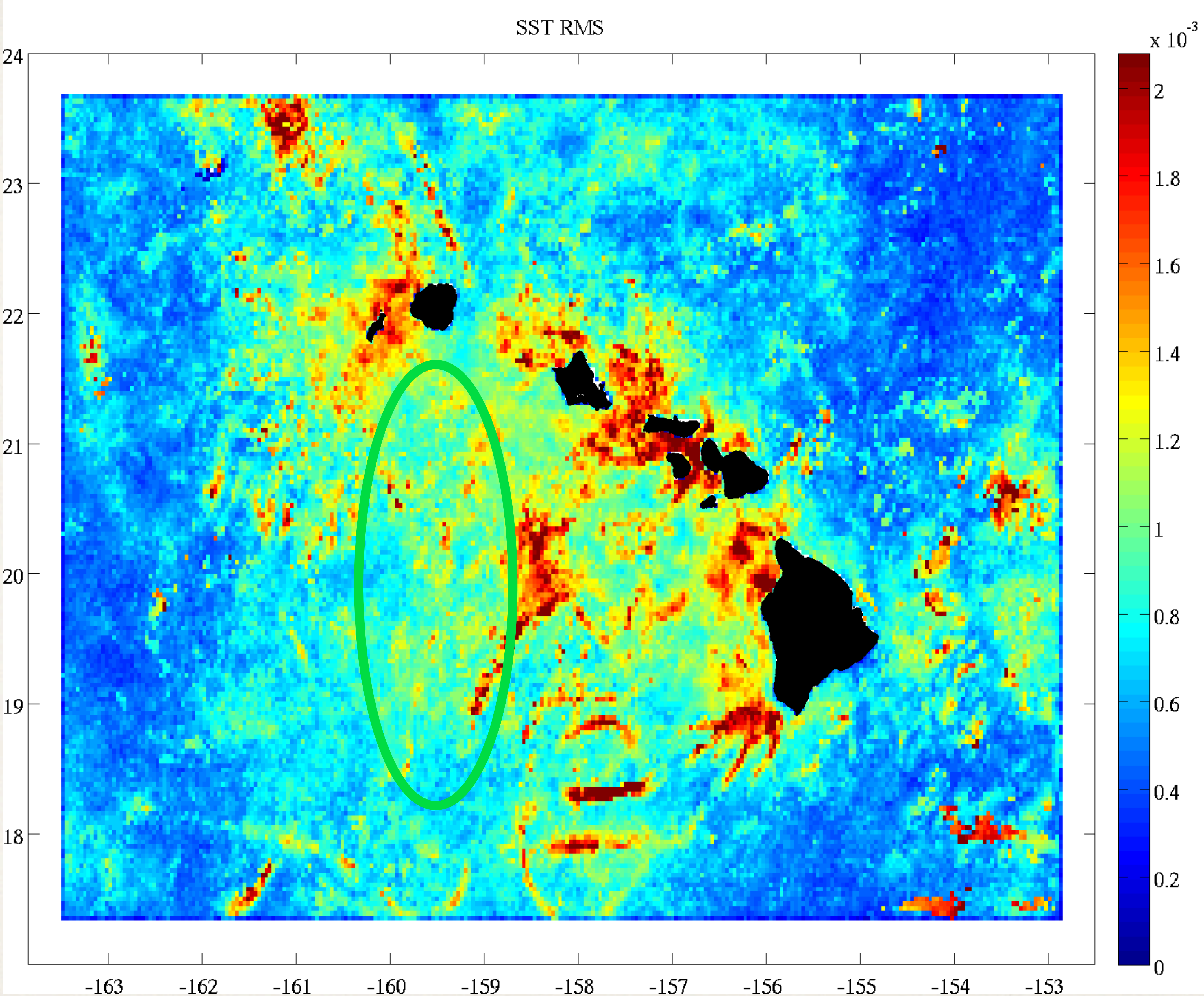


Glider RMS
24
23
22
21
20
19
18
-163
-162
-161
-160
-159
-158
-157
-156
-155
-154
-153
0.2
0.15
0.1
0.05
0



Argo RMS
24
23
22
21
20
19
18
-163
-162
-161
-160
-159
-158
-157
-156
-155
-154
-153
0.022
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002


SSH RMS
x 10^-3
0
1
2
3
4
5
6
7
8
24
23
22
21
20
19
18
-163
-162
-161
-160
-159
-158
-157
-156
-155
-154
-153



SST RMS
x 10^{-3}
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
24
23
22
21
20
19
18
-163
-162
-161
-160
-159
-158
-157
-156
-155
-154
-153

	Temp	Salt	Depth
Glider	9%	5%	0-150m
	32%	42%	150-500m
Profiles	27%	7%	0-150m
	50%	15%	150-500m
		Velocity	
ADCP		21%	0-150m
		59%	150-500m

Innovation Vector: $\mathbf{d} = \mathbf{y} - \mathbf{H}\mathbf{x}_b$

Errors Given By:

$$\epsilon_b = \mathbf{x}_b - \mathbf{x}_t$$
$$\epsilon_y = \mathbf{y} - \mathbf{H}\mathbf{x}_t$$

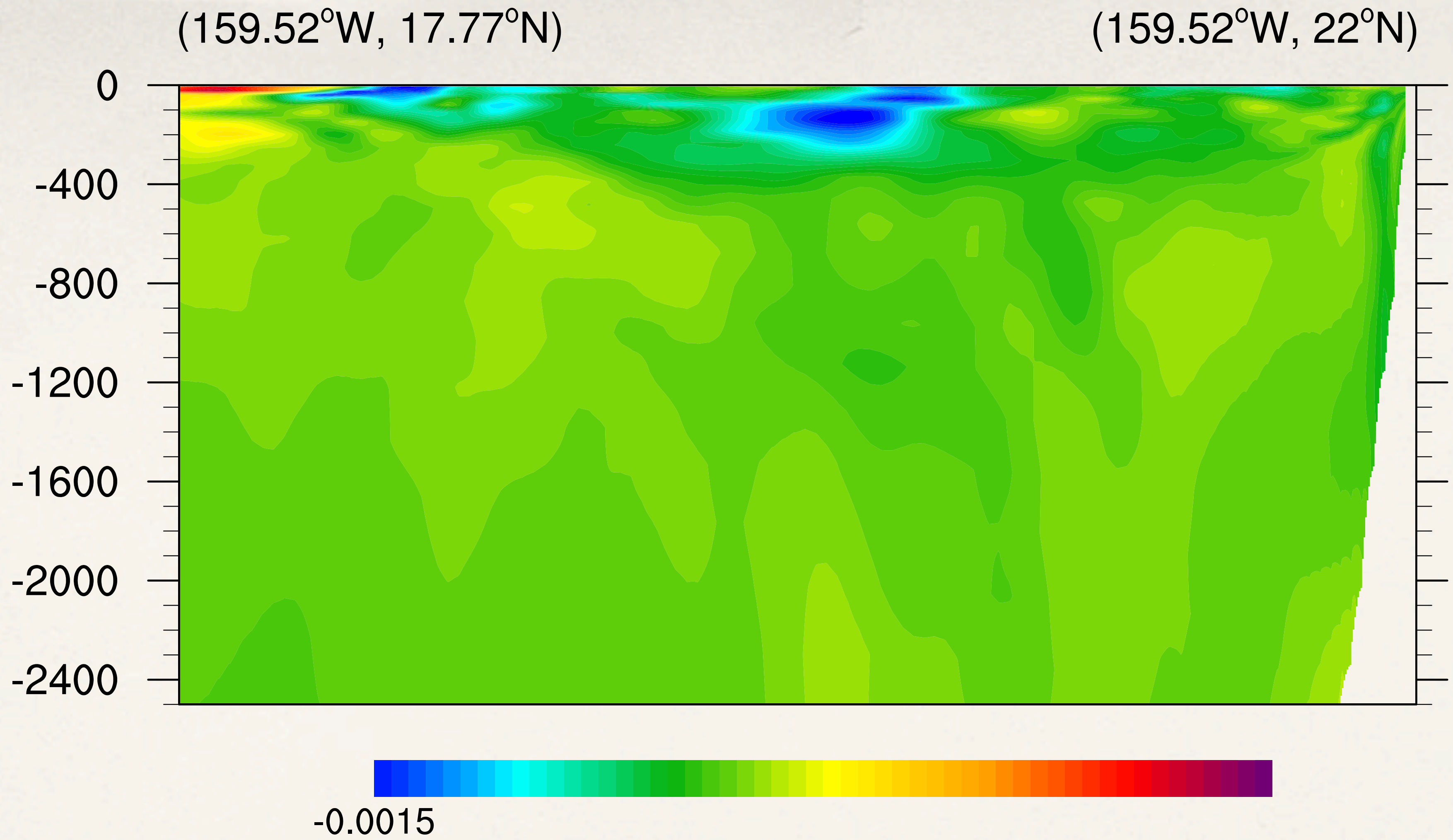
Covariance:

$$\left\langle \mathbf{d}\mathbf{d}^t \right\rangle = \left\langle \left(\epsilon_y - \mathbf{H}\epsilon_b\right)\left(\epsilon_y - \mathbf{H}\epsilon_b\right)^T \right\rangle$$
$$= \mathbf{R} + \mathbf{G}\mathbf{P}\mathbf{G}^T$$

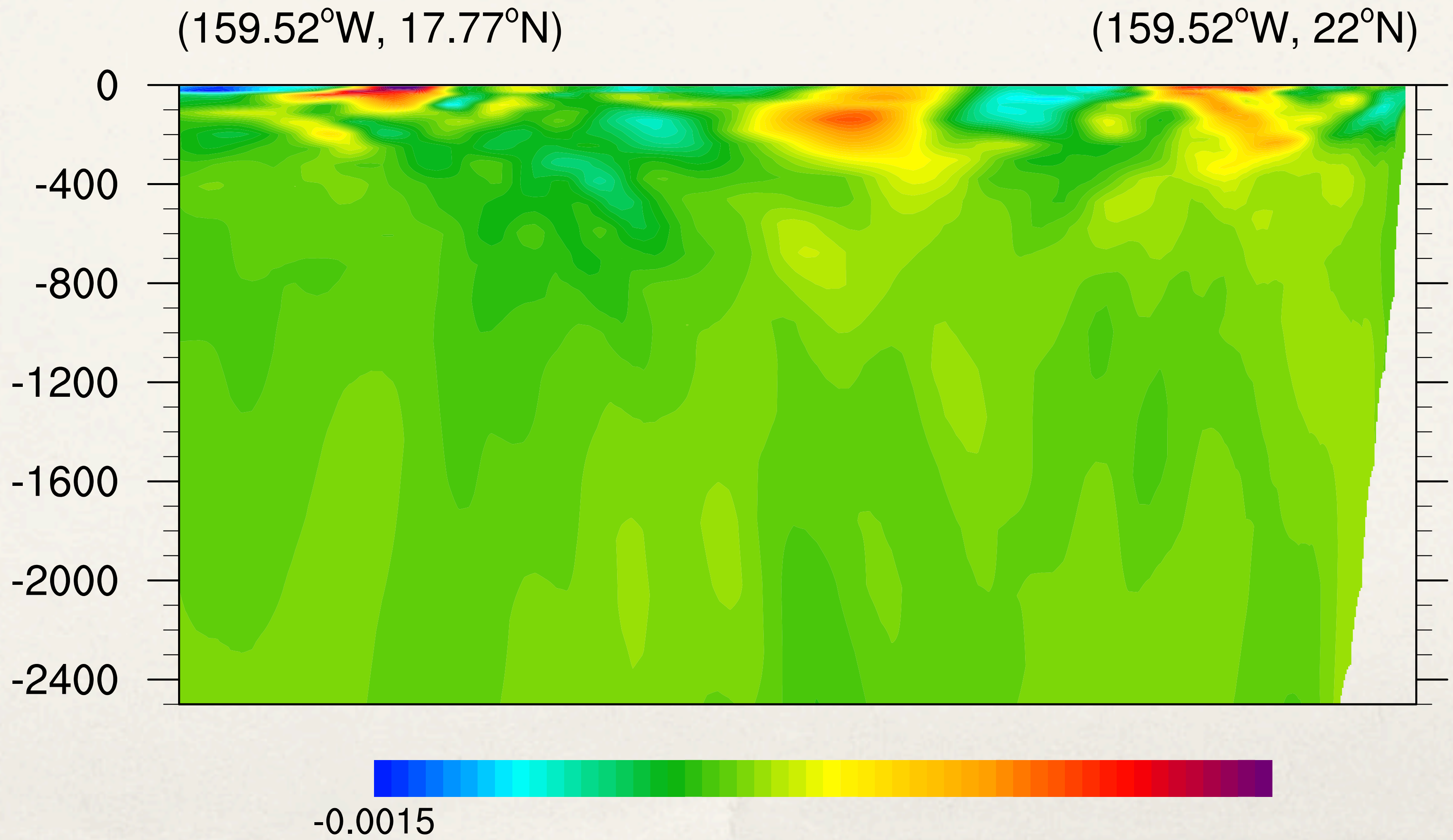
What about a single observation?

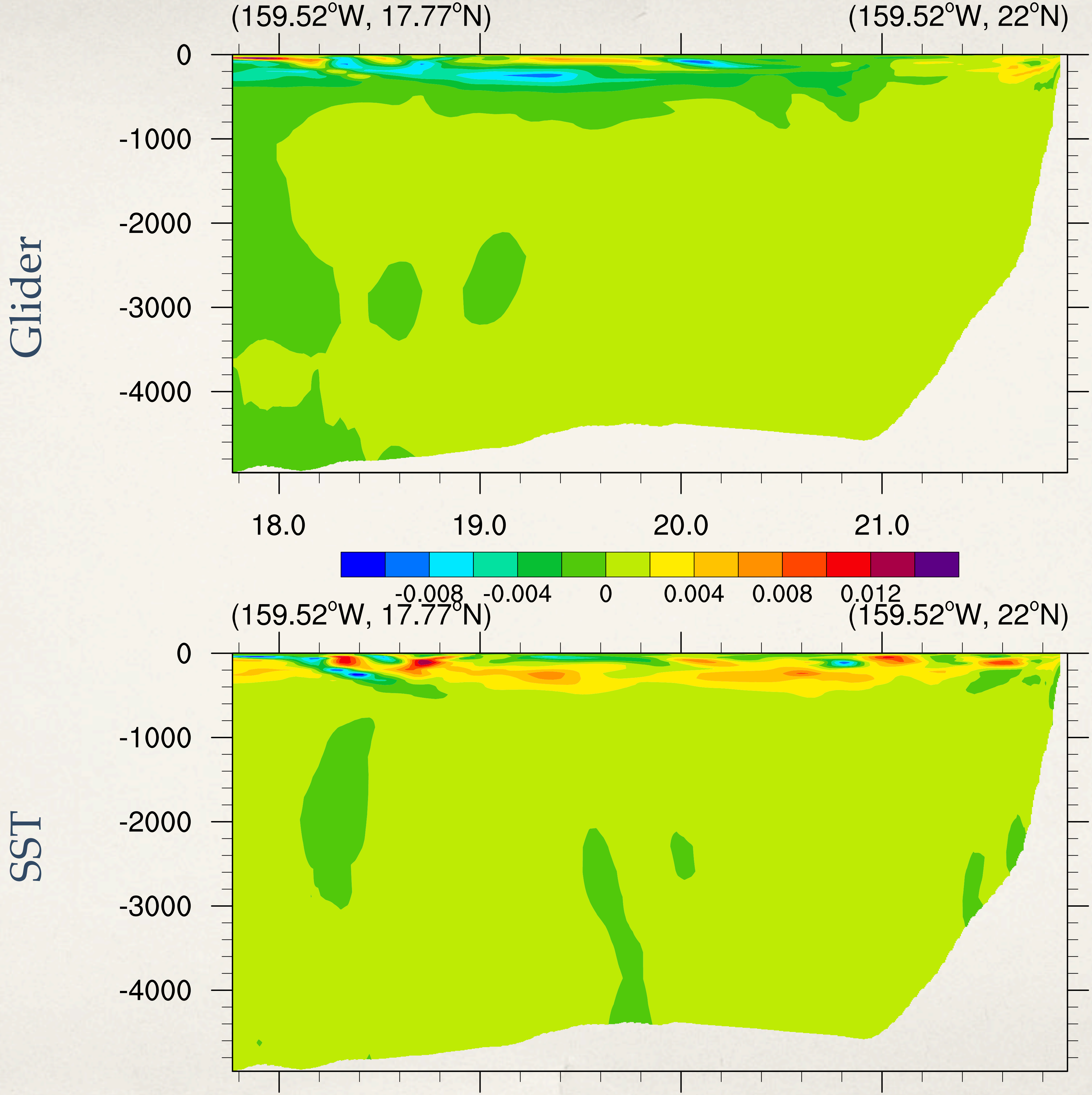
$$\mathbf{r}_m = \mathbf{M}\mathbf{P}\mathbf{G}^T\delta(y_i)$$

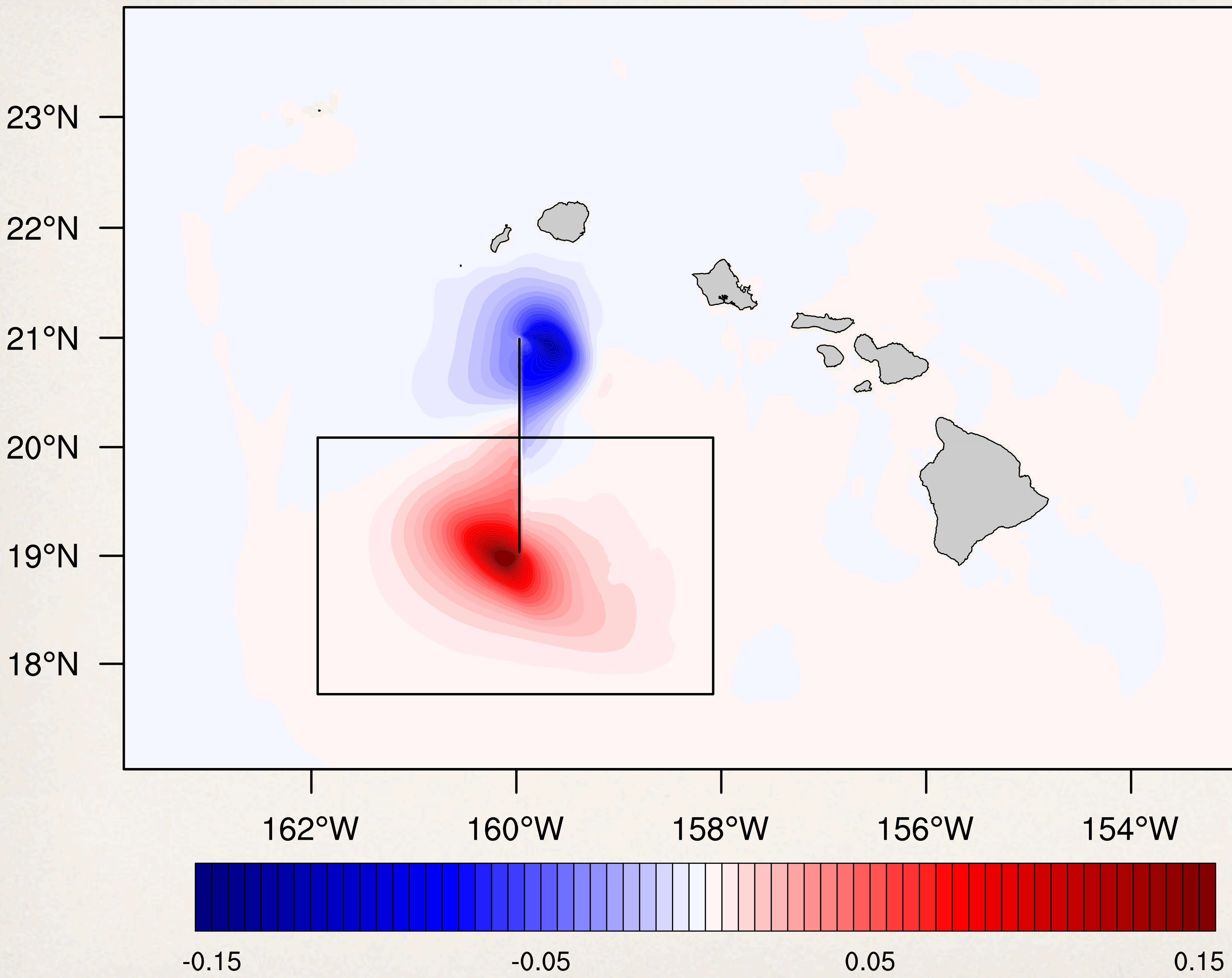
Glider

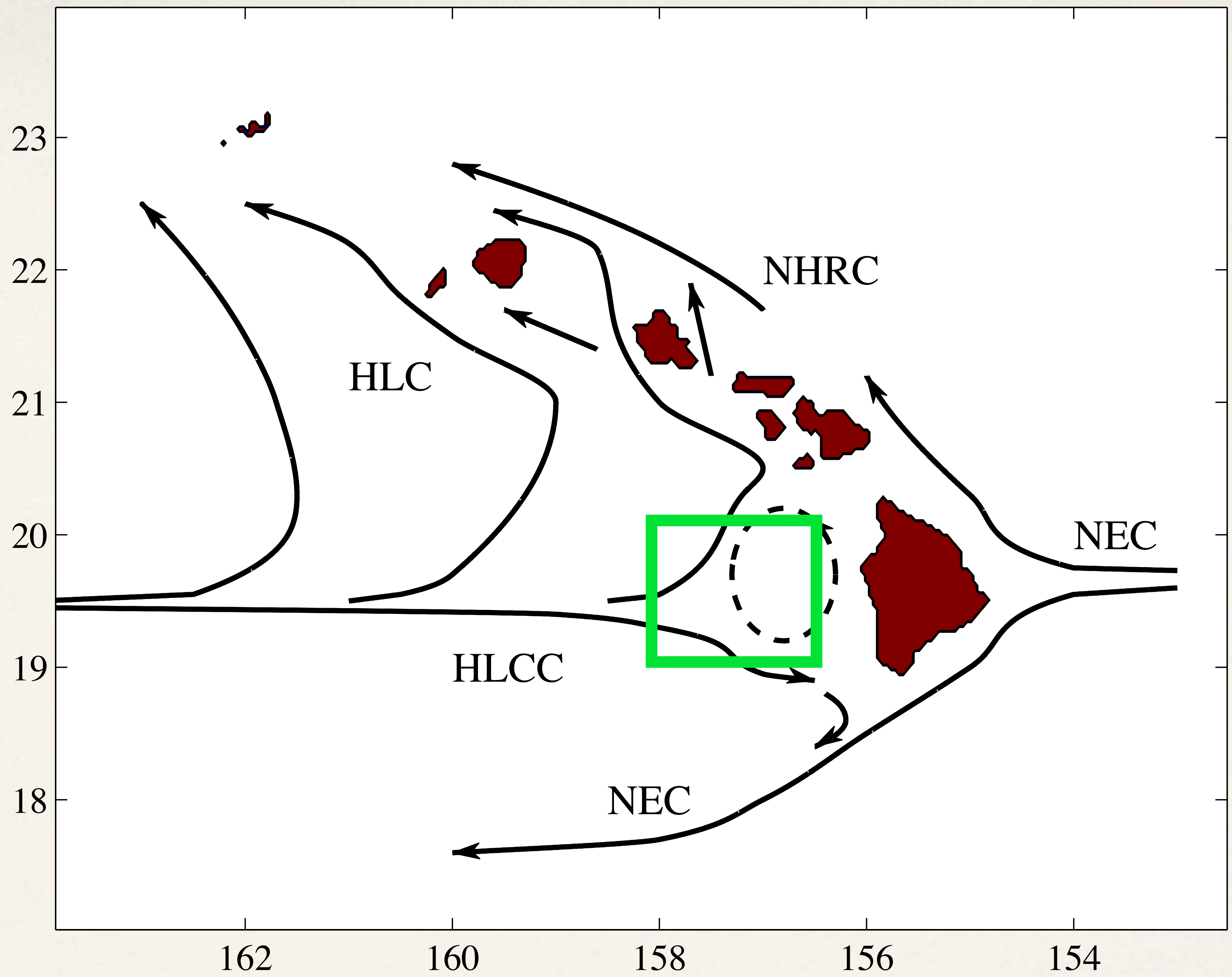

(159.52°W, 17.77°N)
(159.52°W, 22°N)
0
-400
-800
-1200
-1600
-2000
-2400
-0.0015

SST


(159.52°W, 17.77°N)
(159.52°W, 22°N)
0
-400
-800
-1200
-1600
-2000
-2400
-0.0015


Glider
(159.52°W, 17.77°N)
(159.52°W, 22°N)
0
-1000
-2000
-3000
-4000
18.0
19.0
20.0
21.0
-0.008
-0.004
0
0.004
0.008
0.012
SST
(159.52°W, 17.77°N)
(159.52°W, 22°N)
0
-1000
-2000
-3000
-4000

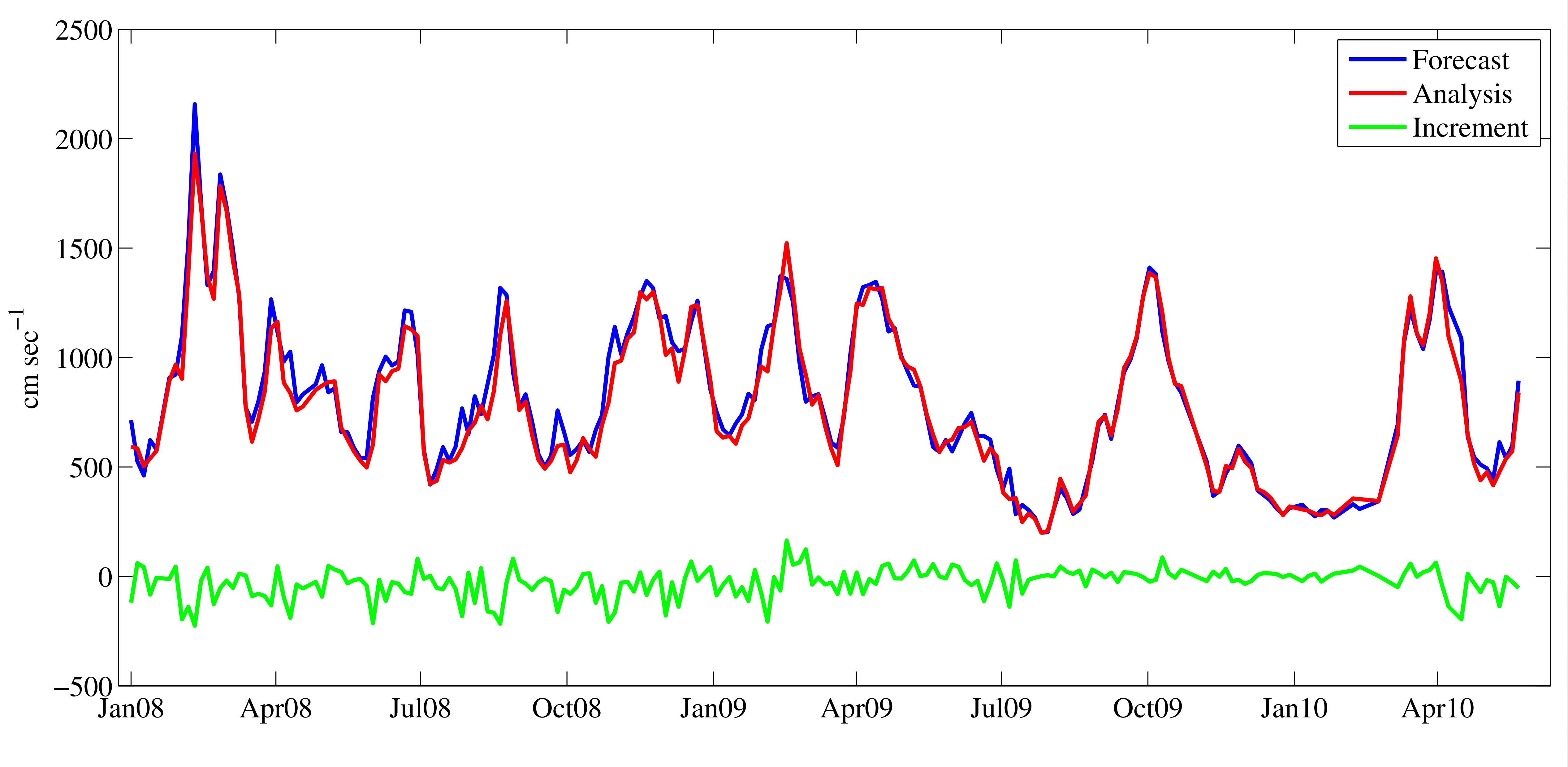

23°N
22°N
21°N
20°N
19°N
18°N
162°W
160°W
158°W
156°W
154°W
-0.15
-0.05
0.05
0.15



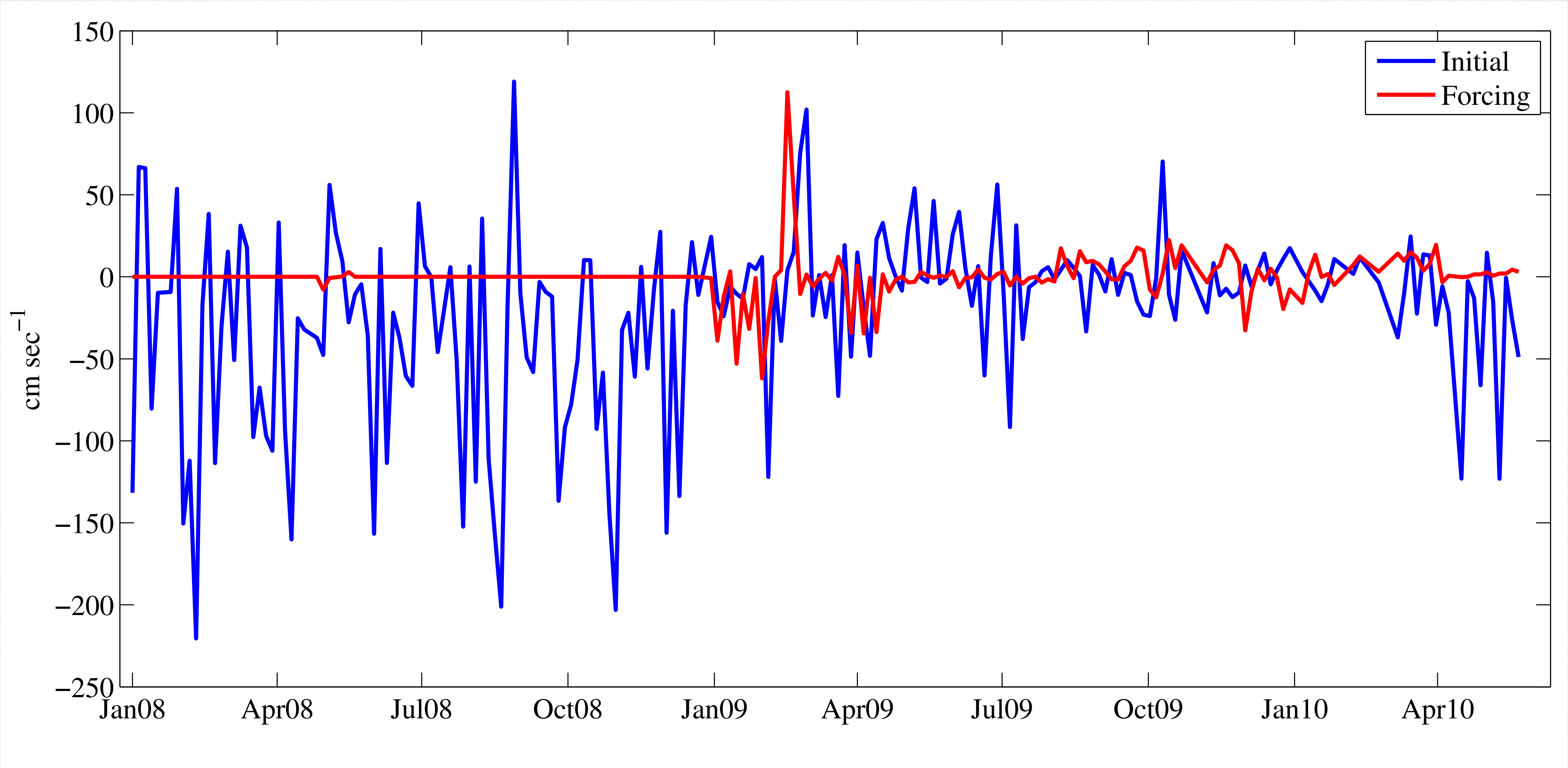
$$\mathcal{J} = \frac{1}{TZS}\int_T\int_S\int_{-Z}^{0}\left(u^2+v^2\right)\mathrm{d}z\mathrm{d}s\mathrm{d}t$$

- Requires higher-order adjoint (Errico, 2007, Gelaro *et al.*, 2007)

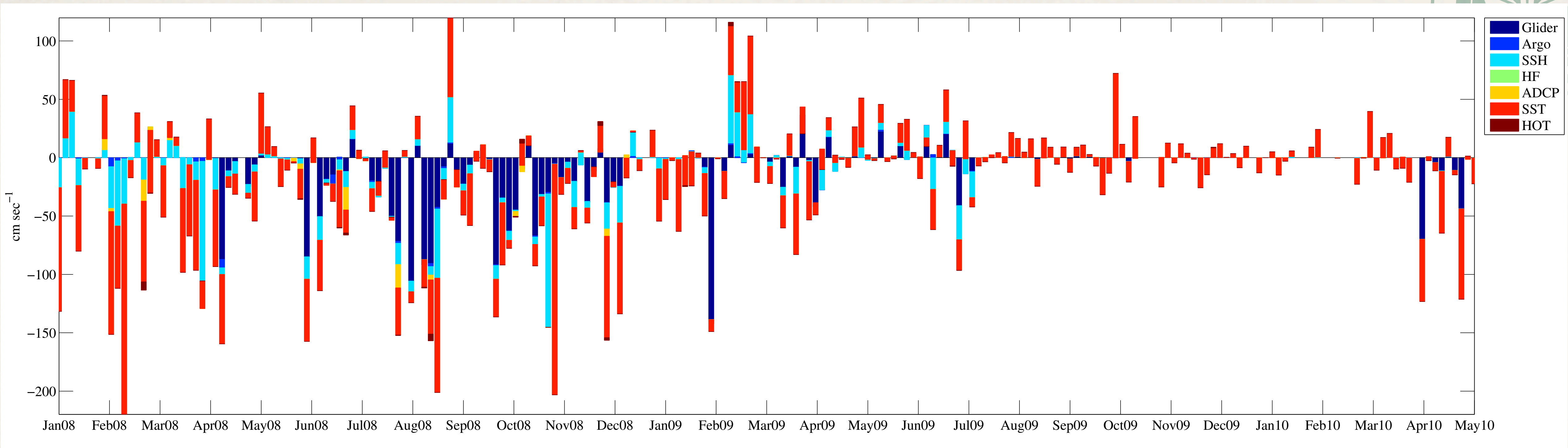
$$\Delta \mathcal{J} = (\mathbf{y} - \mathbf{H}\mathbf{x}_b)^T \mathbf{K}^T \left[\mathbf{M}_b^T \frac{\partial \mathcal{J}}{\partial \mathbf{x}_a} + \mathbf{M}_a^T \frac{\partial \mathcal{J}}{\partial \mathbf{x}_b} \right]$$



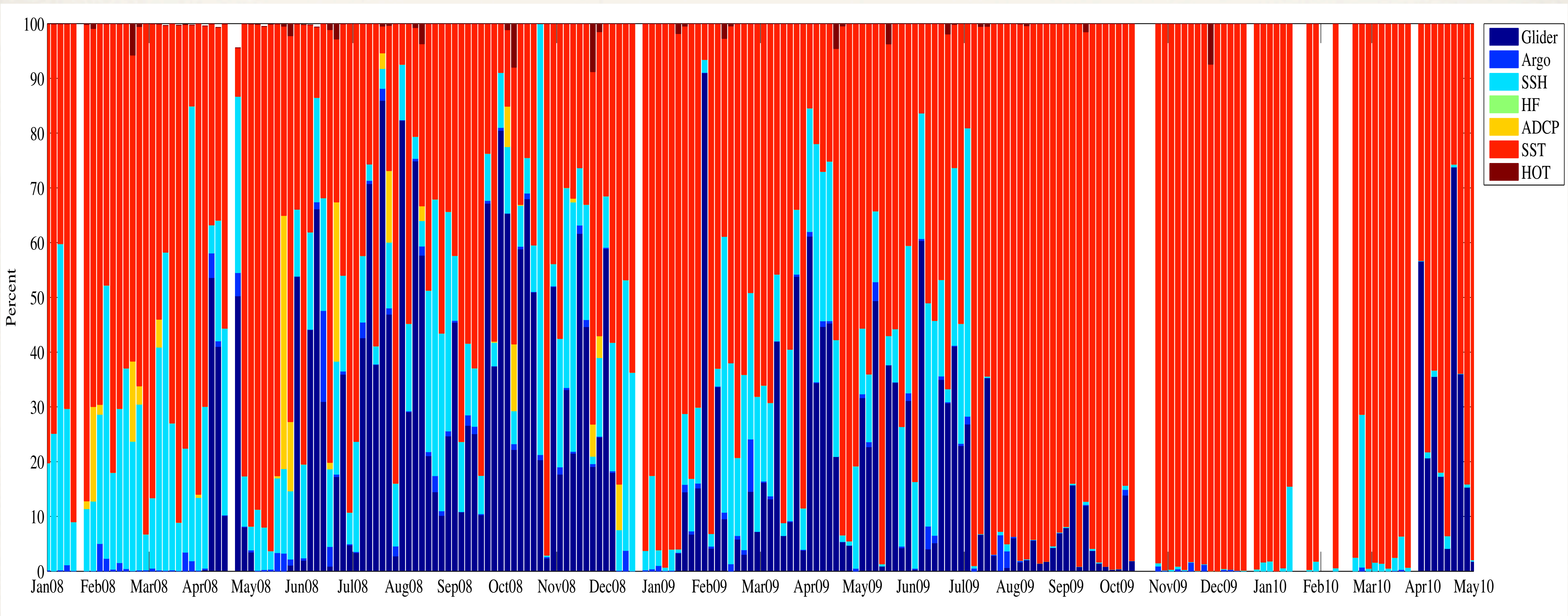
Forecast
Analysis
Increment
cm sec^{-1}
2500
2000
1500
1000
500
0
−500
Jan08
Apr08
Jul08
Oct08
Jan09
Apr09
Jul09
Oct09
Jan10
Apr10



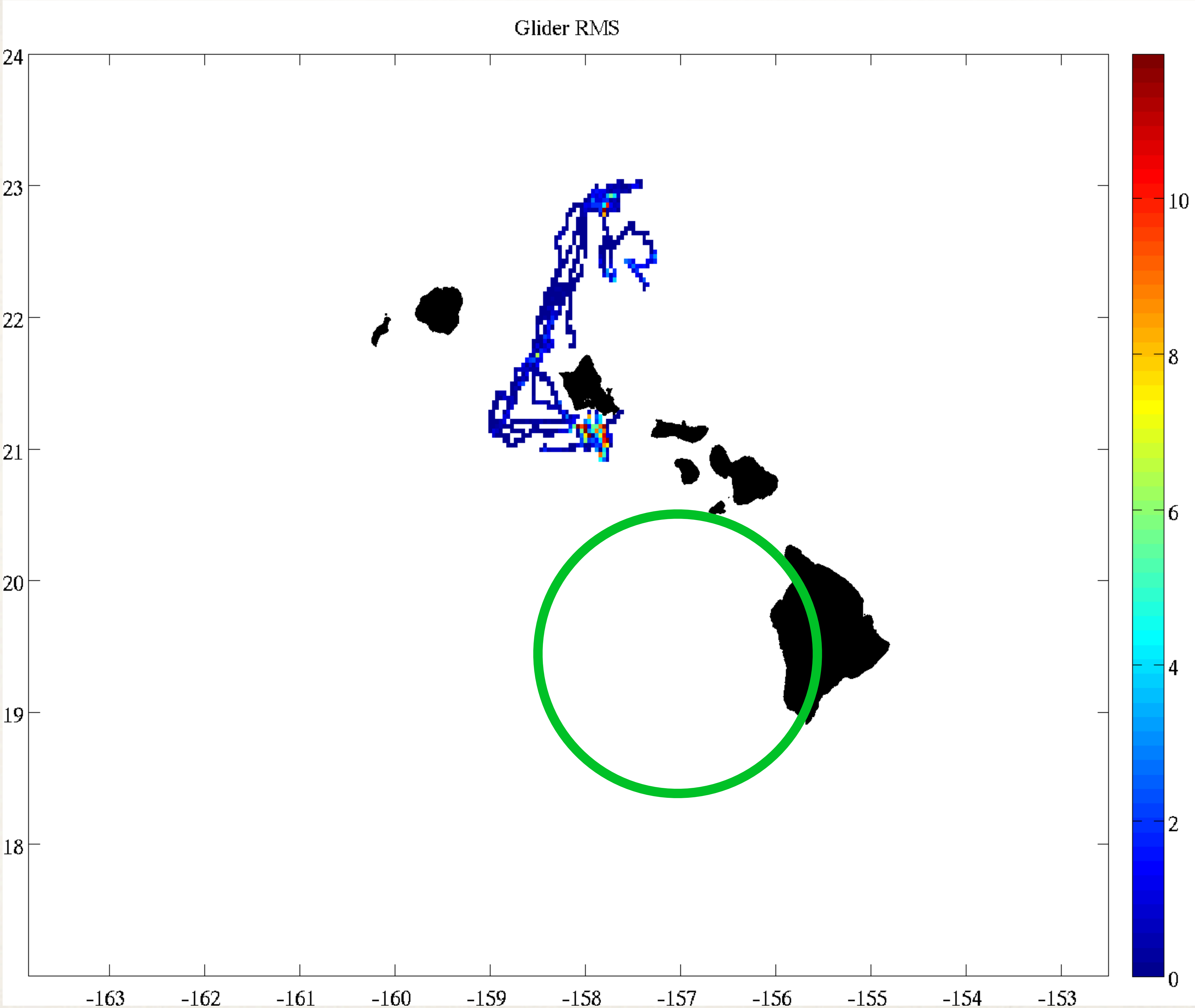
150
100
50
0
−50
−100
−150
−200
−250
cm sec^{-1}
Jan08
Apr08
Jul08
Oct08
Jan09
Apr09
Jul09
Oct09
Jan10
Apr10
Initial
Forcing

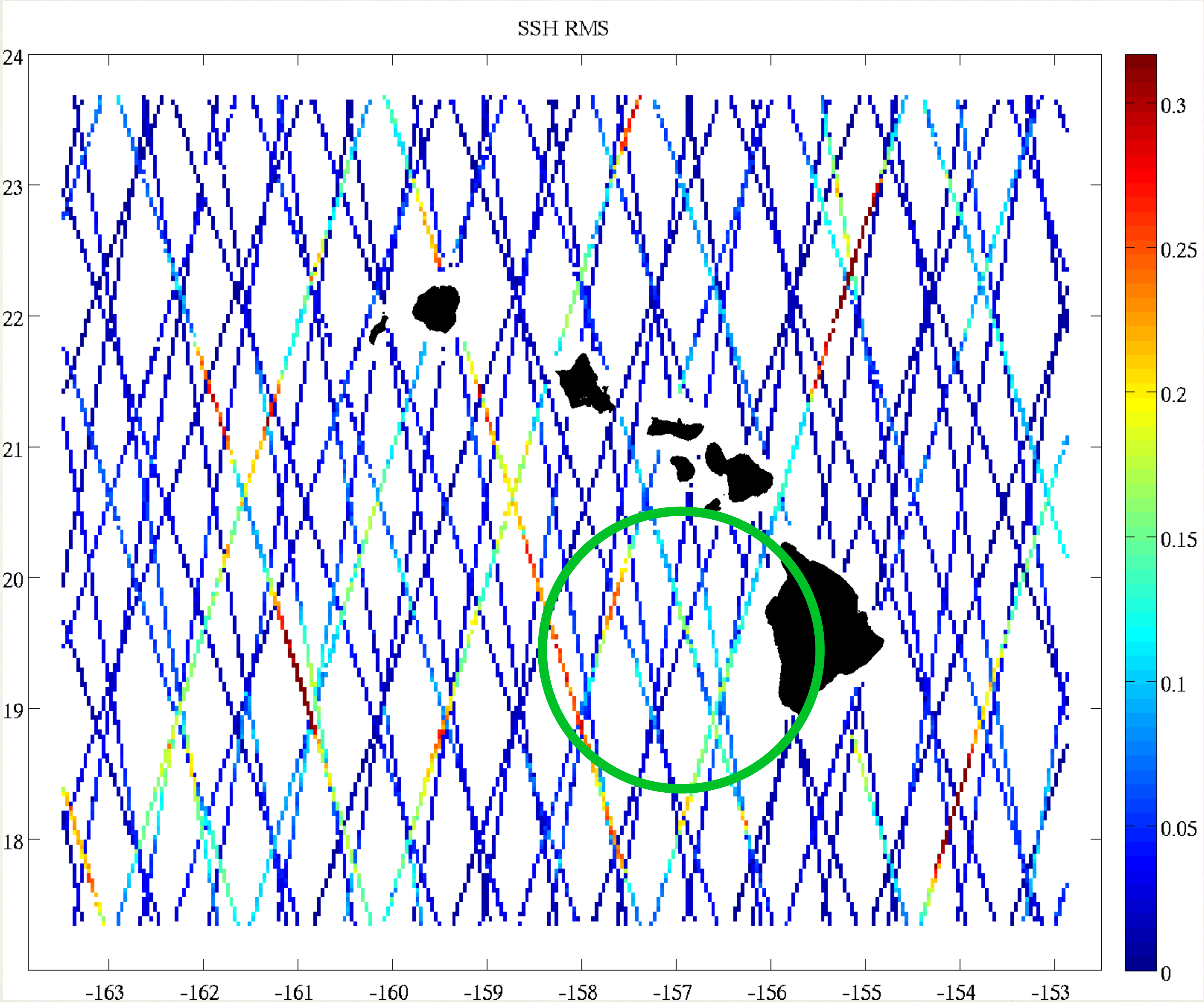


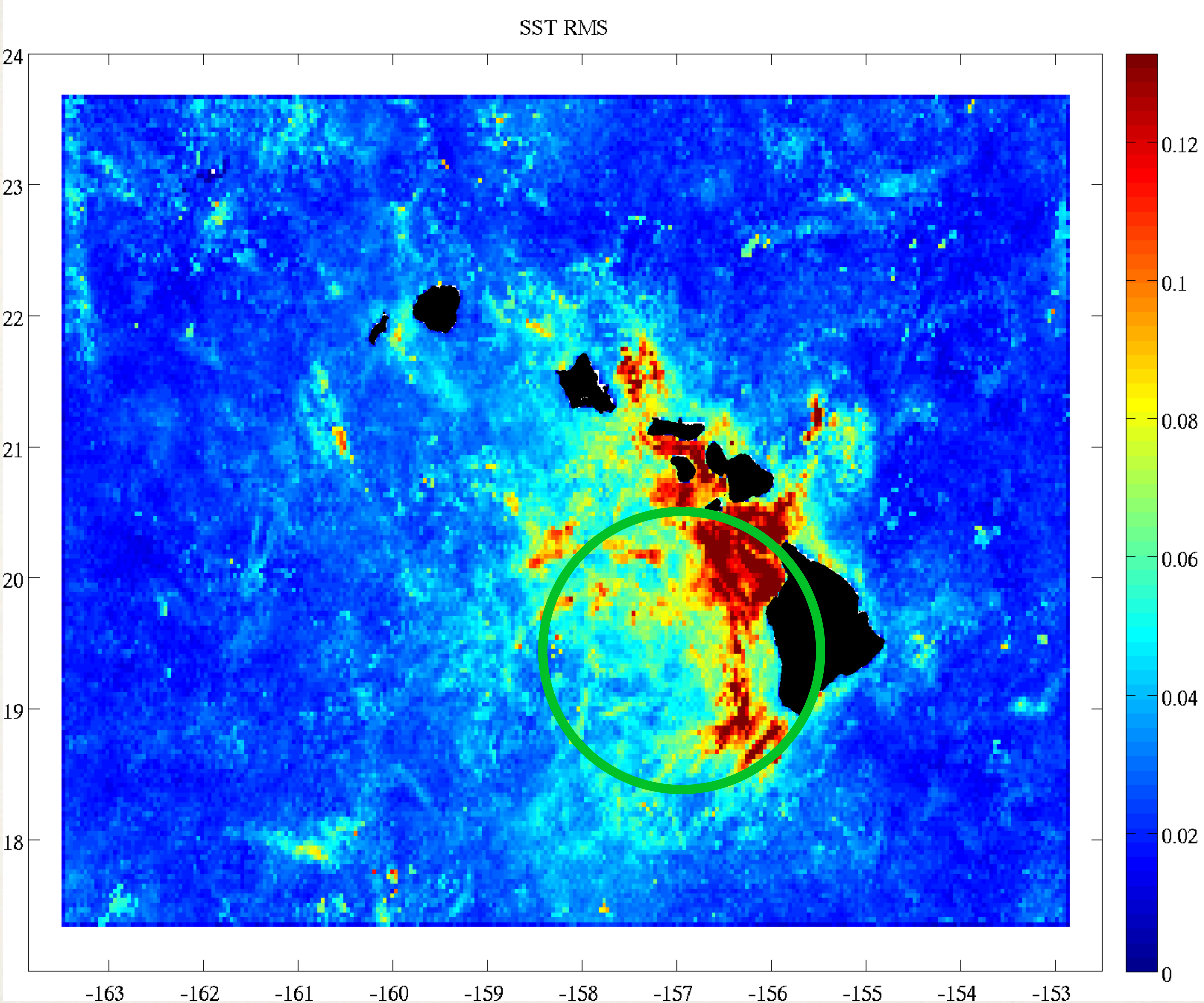
cm sec^{-1}
100
50
0
−50
−100
−150
−200
Jan08 Feb08 Mar08 Apr08 May08 Jun08 Jul08 Aug08 Sep08 Oct08 Nov08 Dec08 Jan09 Feb09 Mar09 Apr09 May09 Jun09 Jul09 Aug09 Sep09 Oct09 Nov09 Dec09 Jan10 Feb10 Mar10 Apr10 May10
Glider
Argo
SSH
HF
ADCP
SST
HOT



Percent
100
90
80
70
60
50
40
30
20
10
0
Jan08 Feb08 Mar08 Apr08 May08 Jun08 Jul08 Aug08 Sep08 Oct08 Nov08 Dec08 Jan09 Feb09 Mar09 Apr09 May09 Jun09 Jul09 Aug09 Sep09 Oct09 Nov09 Dec09 Jan10 Feb10 Mar10 Apr10 May10
Glider
Argo
SSH
HF
ADCP
SST
HOT

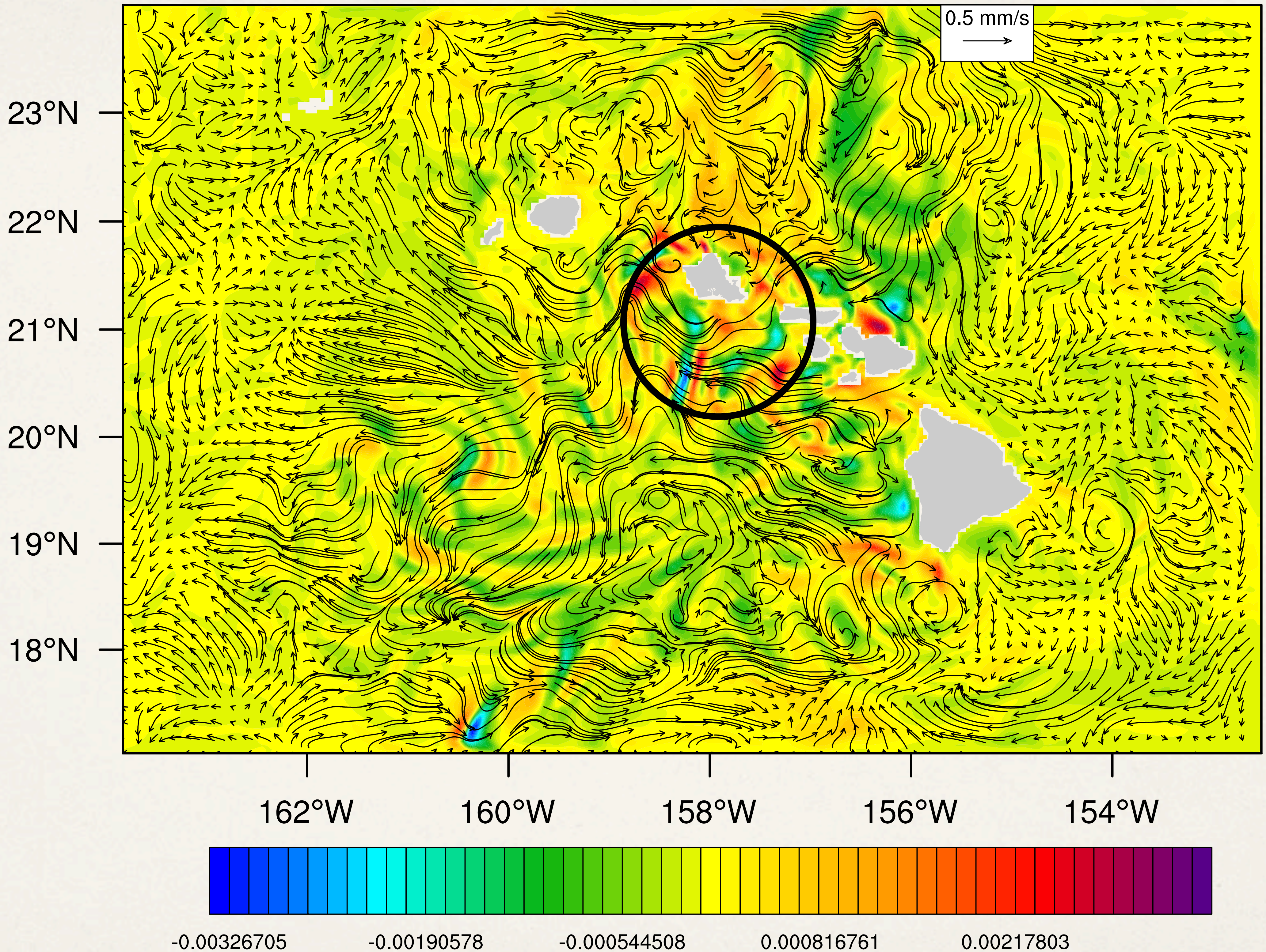

Glider RMS
24
23
22
21
20
19
18
-163
-162
-161
-160
-159
-158
-157
-156
-155
-154
-153
10
8
6
4
2
0


SSH RMS
24
23
22
21
20
19
18
-163
-162
-161
-160
-159
-158
-157
-156
-155
-154
-153
0.3
0.25
0.2
0.15
0.1
0.05
0

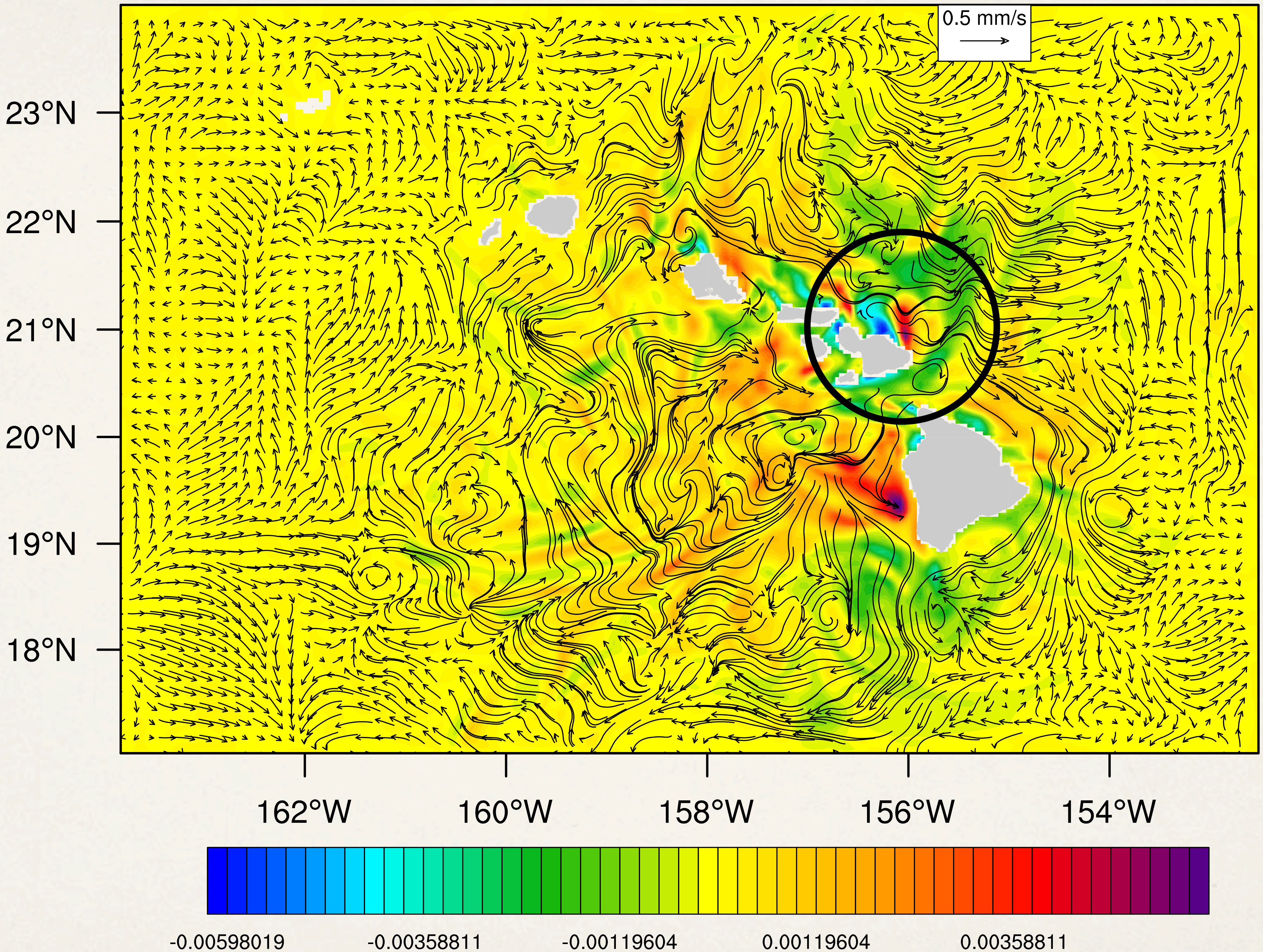


SST RMS
24
23
22
21
20
19
18
-163
-162
-161
-160
-159
-158
-157
-156
-155
-154
-153
0.12
0.1
0.08
0.06
0.04
0.02
0

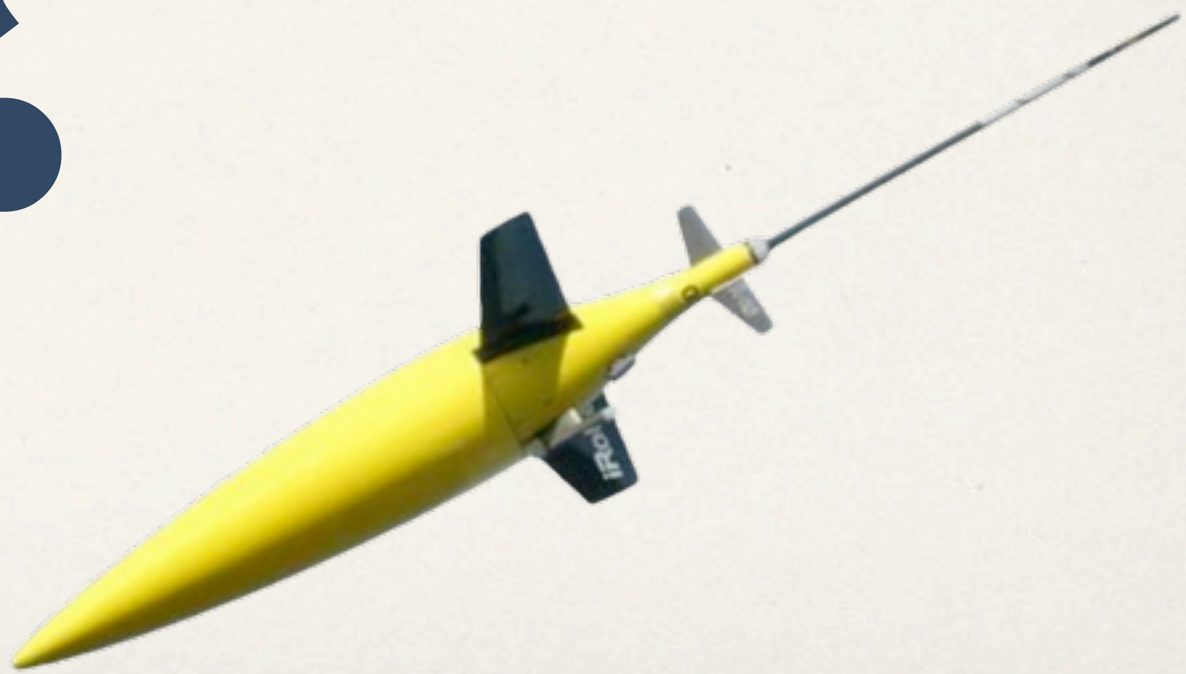
Glider

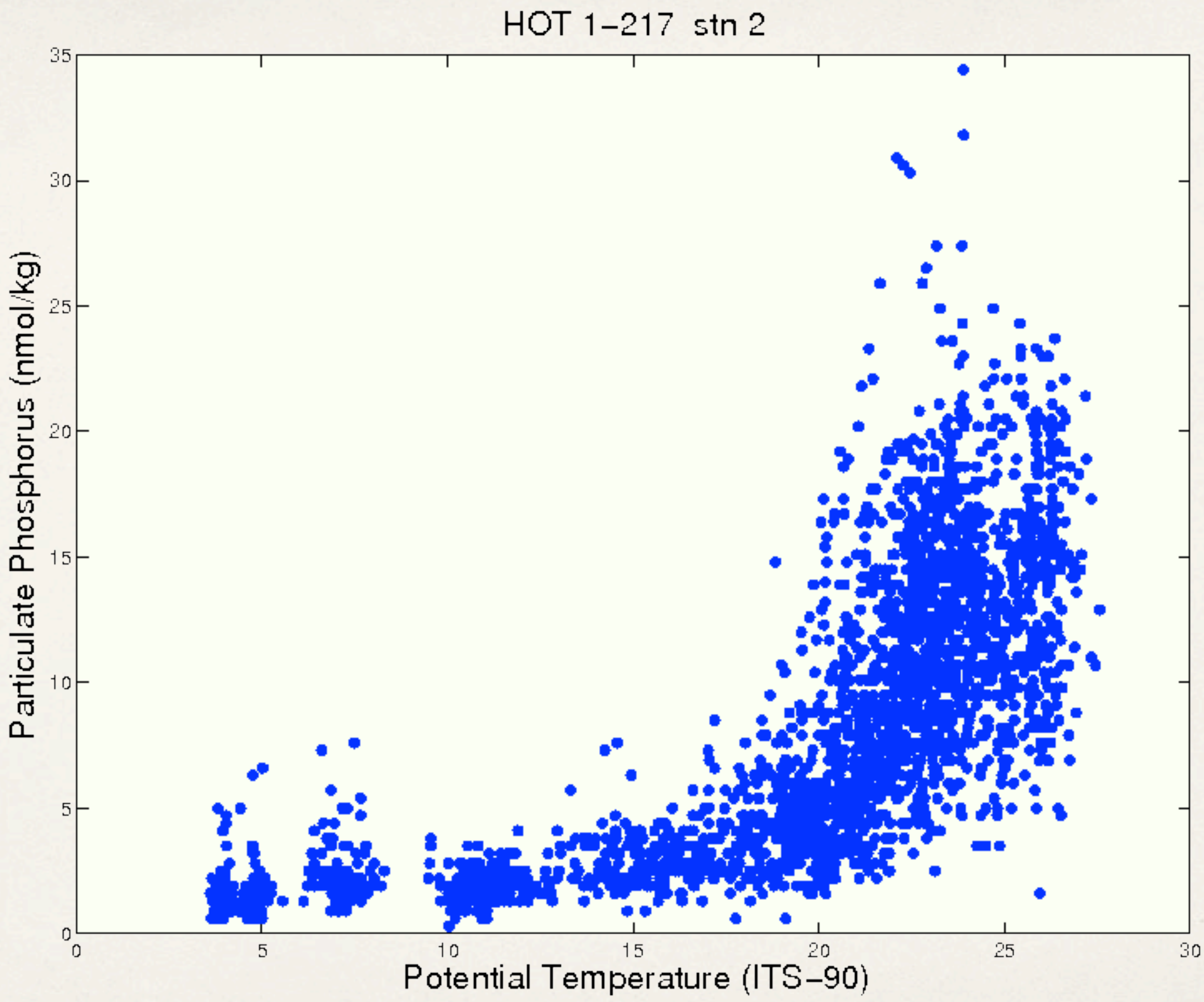

0.5 mm/s
23°N
22°N
21°N
20°N
19°N
18°N
162°W
160°W
158°W
156°W
154°W
-0.00326705
-0.00190578
-0.000544508
0.000816761
0.00217803

SST


0.5 mm/s
23°N
22°N
21°N
20°N
19°N
18°N
162°W
160°W
158°W
156°W
154°W
-0.00598019
-0.00358811
-0.00119604
0.00119604
0.00358811

?




HOT 1–217 stn 2
Particulate Phosphorus (nmol/kg)
Potential Temperature (ITS–90)
0
5
10
15
20
25
30
35