Lateral Boundary Condition (LBC) Control in WRF 4D-Var System

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Outline

Why is LBC Control needed for Regional 4D-Var?

- The Limited Area Model forecasting is a Lateral Boundary Condition (LBC) problem + Initial Condition (IC) problem
- Errico and Vukicevic (1993) show that the forecast error sensitivity with respect to a LBC field might has greater magnitude than for a corresponding IC field in a winter case with MM4 adjoint model.
- Gustafsson and Kallen (1998) indicate that errors in IC as well as in LBC may explain some forecast failures . . . .

Why is LBC Control needed for Regional 4D-Var?

- Phenomena that are observed inside the domain during the later part of the 4D-Var data assimilation window may influence the initial conditions, while being propagated by backward adjoint integration through the lateral boundaries during the early part of the data assimilation window
- Those observed information related to these phenomena may be lost if LBC are not controlled.

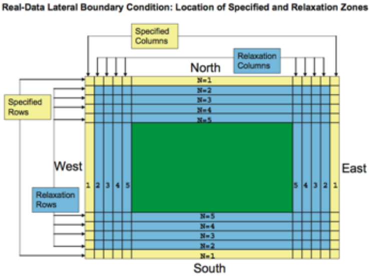
WRF 4D-Var cost function

- Total cost function:
  $J = J_b + J_o + J_c$
- Background term:
  $J_b = \frac{1}{2}(\mathbf{x}(t_0) - \mathbf{x}_b)^T \mathbf{B}^{-1}(\mathbf{x}(t_0) - \mathbf{x}_b)$
- Observational term:
  $J_o = \frac{1}{2}\sum_{k=0}^{K}(H(\mathbf{x}(t_k)) - \mathbf{y}^o(t_k))^T \mathbf{O}^{-1}(H(\mathbf{x}(t_k)) - \mathbf{y}^o(t_k))$
- J_c is the balance term: digital filter

WRF 4D-Var cost function with LBC control

- Total cost function:
  $J = J_b + J_o + J_c + J_{lbc}$
- How to define LBC term?
  $J_{lbc} = \frac{1}{2}(?-?)^T ?^{-1} (?-?)$

WRF specified boundary condition



- WRF lateral boundary : Specified zone +Relaxation zone
- Specified zone : temporal interpolation from an external forecast or analysis
- Relaxation zone : where the model is nudged or relaxed towards the large-scale forecast

Relaxation zone

- Following Davies and Turner (1977):

$$\frac{\partial \mathbf{x}}{\partial t} = F_1(\mathbf{x}_{lbc} - \mathbf{x}) - F_2\Delta^2(\mathbf{x}_{lbc} - \mathbf{x})$$

  where Δ^2 is a 5-point smoothing operator, F_1 and F_2 are (nudging) weighting coefficients depending on the distance to the lateral boundary

- $\mathbf{x}_{lbc}$ is the corresponding boundary value provided by the host model, which is specied in the following form:

$$\mathbf{x}_{lbc}(t) = \mathbf{x}_{lbc}(t_0) + (t - t_0)\frac{\partial \mathbf{x}_{lbc}}{\partial t}$$

WRF LBC

WRF LBC has two parts:

- The starting values of variables within the outer most rows and columns $\mathbf{x}_{lbc}(t_0)$.
- The tendency of variables within the outer most rows and columns $\frac{\partial \mathbf{x}_{lbc}}{\partial t}$, assume the LBC update interval is from t_0 to t_k

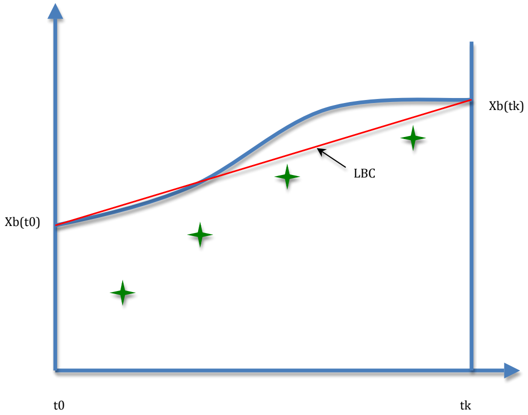
$$\frac{\partial \mathbf{x}_{lbc}}{\partial t} = \frac{\mathbf{x}_{lbc}(t_k) - \mathbf{x}_{lbc}(t_0)}{t_k - t_0}$$

$\mathbf{x}_{lbc}(t_0)$ is extracted from the first guess $\mathbf{x}_b$
$\mathbf{x}_{lbc}(t_k)$ is extracted from the first guess $\mathbf{x}_b$ at t_k

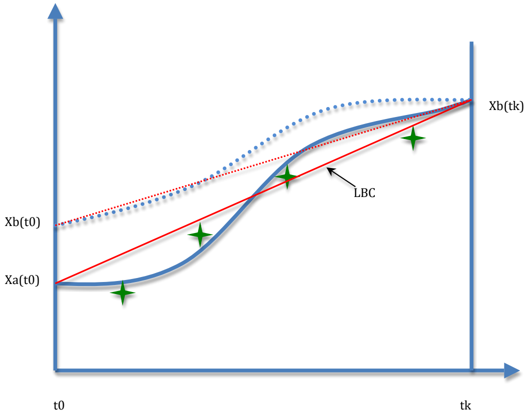
WRF LBC control

- In WRF 4D-Var, if the LBC update interval t_k is equal to the assimilation window (mandatory in practice), the LBC control problem can be reduced to control the model states at both the start and the end of the assimilation window.
- Since it is difficult to only control the model states within the outer most few rows and columns, the whole fields at the end of the assimilation window will be controlled.

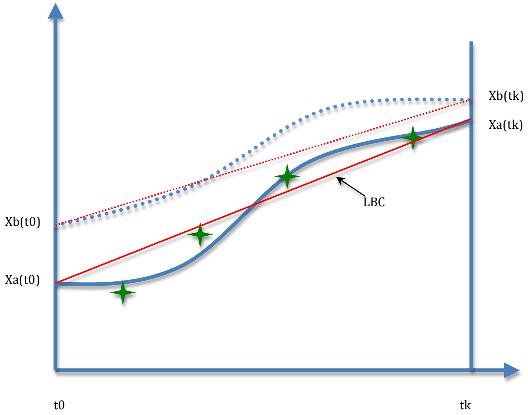
WRF LBC control – first guess



WRF LBC control – without LBC control



WRF LBC control – with LBC control



4D-Var with LBC Control Implementation

Following Kawabata et al. (2007), under context of incremental variational formulation, considering a data assimilation window from time t_0 until time t_k and having $\delta\mathbf{x}(t_0)$ and $\delta\mathbf{x}_{lbc}(t_k)$ as the assimilation control variables

$$J = J_b + J_o + J_c + J_{lbc}$$

$$\begin{aligned} J_{lbc} &= \frac{1}{2}(\mathbf{x}(t_k) - \mathbf{x}_b(t_k))^T \mathbf{B}^{-1}(\mathbf{x}(t_k) - \mathbf{x}_b(t_k)) \\ &= \frac{1}{2}\delta\mathbf{x}(t_k)^T \mathbf{B}^{-1}\delta\mathbf{x}(t_k) \end{aligned}$$

J_{lbc} is the J_b at the end of the assimilation window

lateral boundary control is obtained through

$$\frac{\partial \delta\mathbf{x}_{lbc}}{\partial t} = \frac{\delta\mathbf{x}(t_k) - \delta\mathbf{x}(t_0)}{t_k - t_0}$$

LBC perturbation

The quantities needed for the LBC of the tangent linear model are given by

$$\delta \mathbf{x}_{lbc}(t_0) = \delta \mathbf{x}(t_0)$$

$$\frac{\partial \delta \mathbf{x}_{lbc}}{\partial t} = \frac{\delta \mathbf{x}_{lbc}(t_k) - \delta \mathbf{x}(t_0)}{t_k - t_0}$$

LBC adjoint

The LBC for the adjoint model, $\mathbf{x}_{lbc}^{AD}$ and $(\frac{\partial \mathbf{x}_{lbc}}{\partial t})^{AD}$, will be initialized with zeroes at the end of the data assimilation window (time t_k). After the backwards integration of the adjoint model to time t_0, the adjoint control variables (or the gradients) can be obtained from:

$$\mathbf{x}_{ic}^{AD}(t_0) = \mathbf{x}_{ic}^{AD}(t_0) + \mathbf{x}_{lbc}^{AD}(t_0) - \frac{1}{t_k - t_0}(\frac{\partial \mathbf{x}_{lbc}}{\partial t})^{AD}$$

$$\mathbf{x}_{lbc}^{AD}(t_k) = \frac{1}{t_k - t_0}(\frac{\partial \mathbf{x}_{lbc}}{\partial t})^{AD}$$

where $\mathbf{x}_{ic}^{AD}(t_0)$ denotes the inner domain adjoint model model variable as provided at the initial time t_0.

- Analysis increments due to a single observation produced by a 4D-Var system describe how the tangent linear and adjoint model propagate the observational information.
- Putting a simulated 500hPa temperature observation at 6h (at the end of the assimilation window) close to inflow lateral boundary (within relaxation zone), we expect some information will be propagated from the inflow lateral boundary.

500hPa θ analysis increments at 0 h

$+$ is the location of the observation at 6h

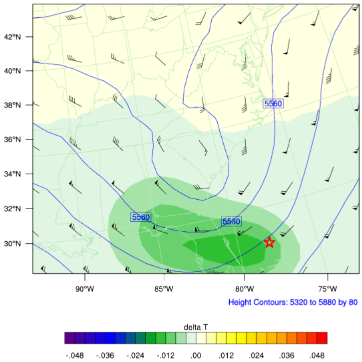


Figure: w/o LBC control

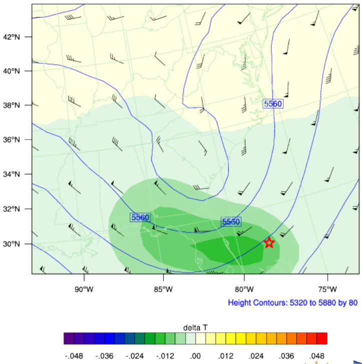
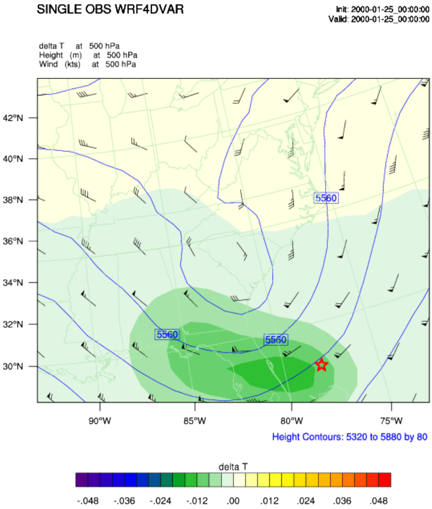
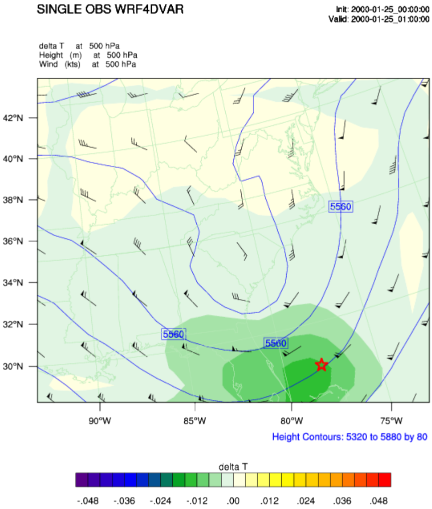


Figure: with LBC control

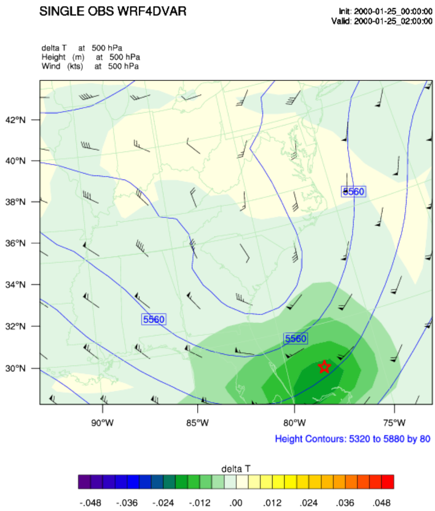
500hPa θ increments at 00h with LBC control



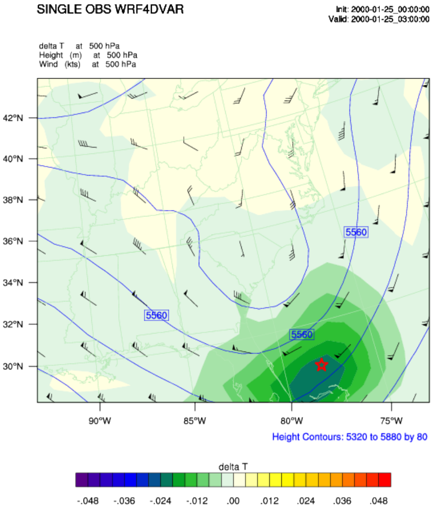
500hPa θ increments at 01h with LBC control



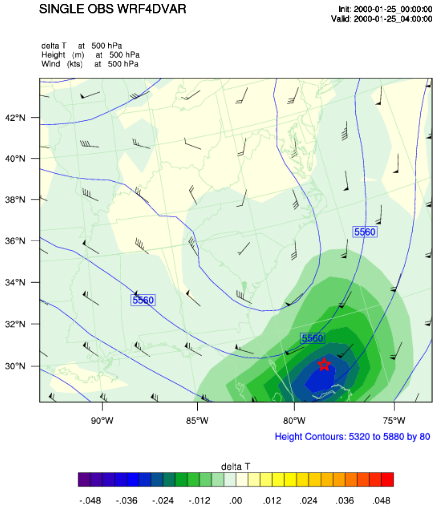
500hPa θ increments at 02h with LBC control



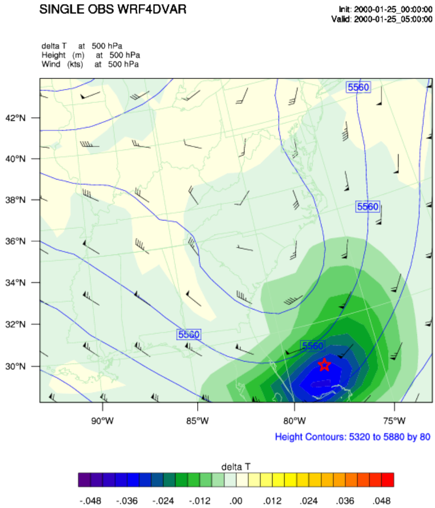
500hPa θ increments at 03h with LBC control



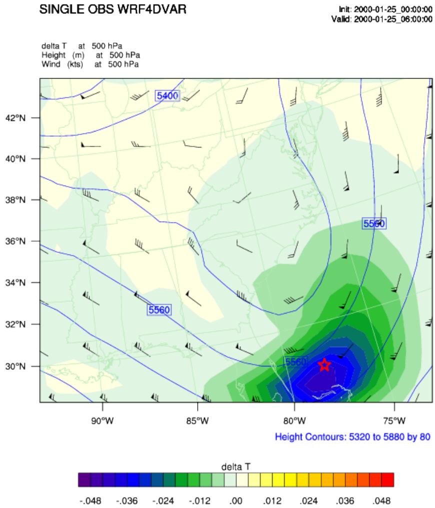
500hPa θ increments at 04h with LBC control



500hPa θ increments at 05h with LBC control

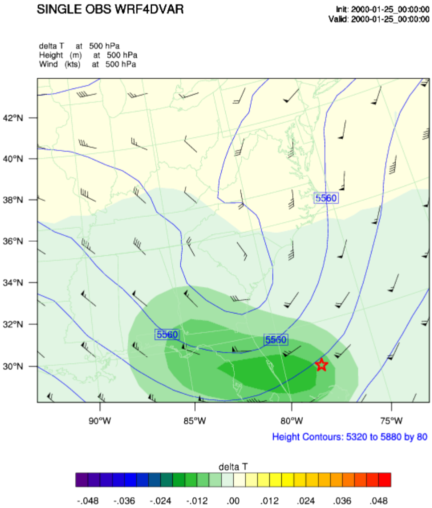


500hPa θ increments at 06h with LBC control

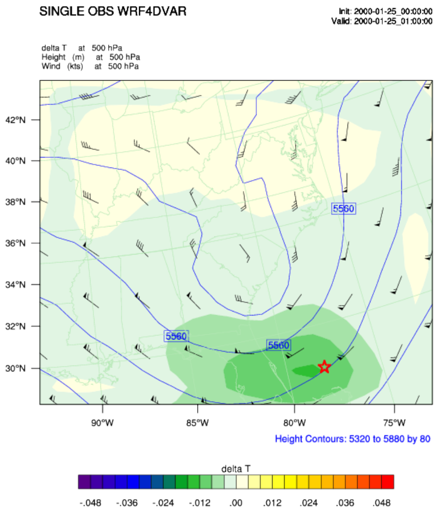


- Clearly flow-dependent increments is located upstream the observation in IC.
- The boundary conditions at both the start and end of the assimilation window are changed.
- The analysis increment of -0.48K at the observation location on 6h.
- The LBC contributes the information to propagate ! ?

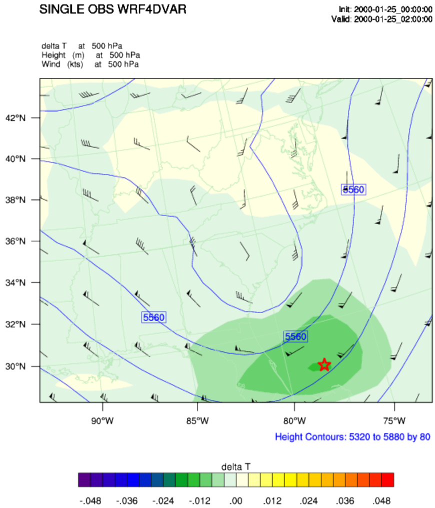
500hPa θ increments at 00h w/o LBC control



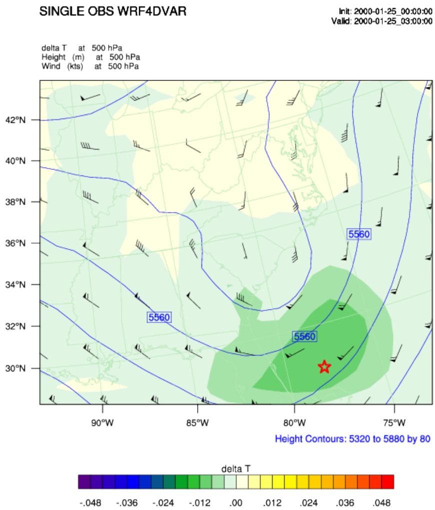
500hPa θ increments at 01h w/o LBC control



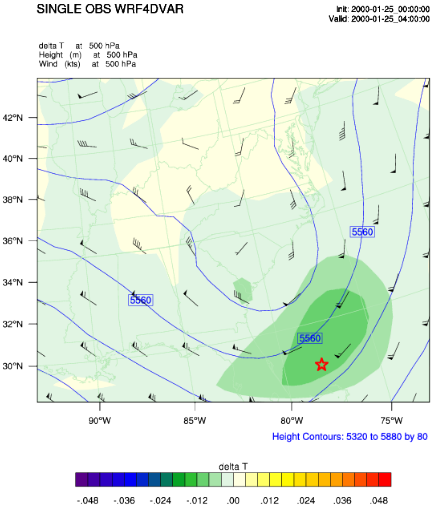
500hPa θ increments at 02h w/o LBC control



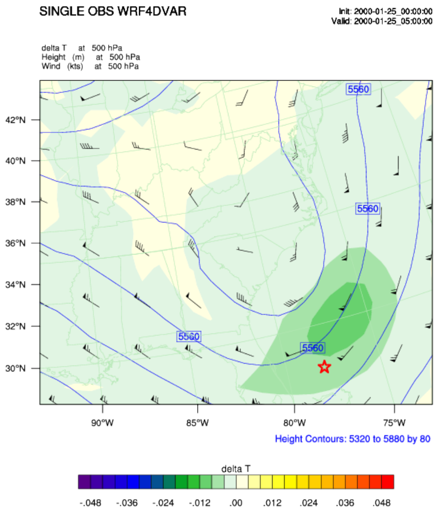
500hPa θ increments at 03h w/o LBC control



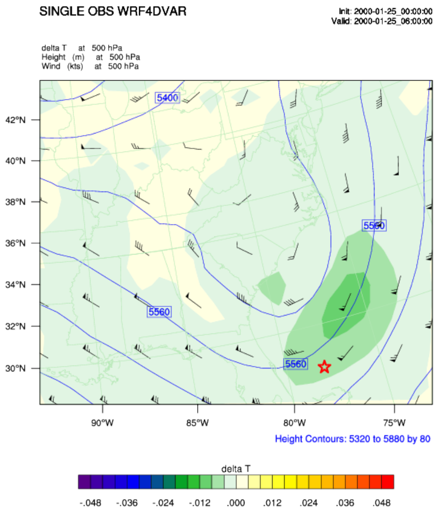
500hPa θ increments at 04h w/o LBC control



500hPa θ increments at 05h w/o LBC control



500hPa θ increments at 06h w/o LBC control



- Clearly flow-dependent increments is located upstream the observation in IC.
- Only the boundary conditions at the start of the assimilation window is changed and which at the end of window is kept unchanged
- The analysis increment of -0.04K at the observation location on 6h.
- The main analysis increments pass away the observation on 6h

Summary and Discussion

- Single observation experiments confirm the implementation of LBC control in WRF 4D-Var
- Inclusion of LBC control in 4D-Var helps to obtain an analysis which fits the observation better
- The preliminary real observation experiments with LBC control don't show noticeable improvement, but Gustafsson (personal communication) implemented LBC control with same way in the HIRLAM 4D-Var system and the one-month operational experiment show the positive impact on verification score.

Summary and Discussion

- A common question regarding to the correlation between control variables at the start and the end of the window, Gustafsson (personal communication) confirmed that the correlation is very small in HIRLAM , which means the 4D-Var with LBC should be well conditioned.
- Our experiments with LBC control don't show more iterations are needed to converge compared to those without LBC control .